UNIVERSITY OF TEXAS AT AUSTIN

HW Assignment 4

Exchange options. Bull/Bear spreads. Properties of European call/put prices.

4.1. **Exchange options.** Provide your **complete solution** to the following problem:

Problem 4.1. (5 points) **The minimum option**

Let $\mathbf{S} = \{S(t), t \geq 0\}$ and $\mathbf{Q} = \{Q(t), t \geq 0\}$ denote the prices of two risky assets. The payoff of the *minimum option* is given by

$$V_{min}(T) = \min(S(T), Q((T)).$$

Propose a replicating portfolio consisting of prepaid forward contracts on $\mathbf{S}$ and/or $\mathbf{Q}$, and exchange options on $\mathbf{S}$ and $\mathbf{Q}$.

Problem 4.2. (5 points) Let our market model include two continuous-dividend-paying stocks whose time$-t$ prices are denoted by $S(t)$ and $Q(t)$ for $t \geq 0$. The current stock prices are $S(0) = 160$ and $Q(0) = 80$. The dividend yield for the stock S is $\delta_S = 0.06$ and the dividend yield for the stock Q is $\delta_Q = 0.03$.

The price of an exchange option giving its bearer the right to forfeit one share of Q for one share of S in one year is given to be \$11.

Find the price of a maximum option on the above two assets with exercise date in a year. Remember that the payoff of the maximum option is $\max(S(1), Q(1))$.

Provide your **final answer only** to the following problems:

Problem 4.3. (2 points)

Exchange options are options where the underlying asset is an exchange rate. *True or false?*

Problem 4.4. (2 points) Consider two European exchange options both with exercise date T, one that allows you to exchange a share of asset S for a share of asset Q, and another one that allows you to forfeit a share of asset Q and obtain a share of asset S in return. Assume that neither asset pays any dividends.

As usual $V_{EC}(0, X(0), Y(0))$ stands for the time-0 price of the exchange call with the underlying asset X and the strike asset Y.

Then, in our usual notation,

$$V_{EC}(0, \mathbf{S}, \mathbf{Q}) - V_{EC}(0, \mathbf{Q}, \mathbf{S}) = S(0) - Q(0). \tag{4.1}$$

Problem 4.5. (5 points) Assume that the continuously compounded interest rate equals 0.10.

Stock S has the current price of $S(0) = 70$ and does not pay dividends. Stock Q has the current price of $Q(0) = 65$ and it pays continuous dividends at the rate of 0.04.

An exchange option gives its holder the right to give up one share of stock Q for a share of stock S in exactly one year. The price of this option is \$11.50.

Another exchange option gives its holder the right to give up one share of stock S for a share of stock Q in exactly one year. Find the price of this option.

(a) About \$3.95
(b) About \$11.10
(c) About \$12.00
(d) About \$14.25
(e) None of the above.

Problem 4.6. (2 points) Consider two exchange options, one that allows you to exchange a share of asset S for a share of asset Q, and another one that allows you to forfeit a share of asset Q and obtain a share of asset S in return.
Then, the prepaid forward prices of the two assets are the same if and only if the two exchange options have the same price.

Problem 4.7. (2 points) Consider two exchange options, one that allows you to exchange a share of asset S for a share of asset Q, and another one that allows you to forfeit a share of asset Q and obtain a share of asset S in return. Assume neither of the two stocks pays dividends.
Then, the current spot prices of the two assets are the same if and only if the two exchange options have the same price.

Problem 4.8. (2 pts) Consider two European exchange options both with exercise date T, one that allows you to exchange a share of asset S for a share of asset Q, and another one that allows you to forfeit a share of asset Q and obtain a share of asset S in return.
On the other hand, consider the maximum option with the payoff

$$V_{max}(T) = \max(S(T), Q(T)),$$

and the minimum option with the payoff

$$V_{min}(T) = \min(S(T), Q(T)).$$

Then, in our usual notation,

$$V_{EC}(0, \mathbf{S}, \mathbf{Q}) + V_{EC}(0, \mathbf{Q}, \mathbf{S}) = V_{max}(0) + V_{min}(0).$$

4.2. **Monotonicity of option prices.**

Problem 4.9. (2 pts) Suppose that the European options with the same maturity and the same underlying assets have the following prices:

(1) a 50−strike call costs \$9;
(2) a 55−strike call costs \$10;

Then, some of the *monotonicity* conditions for no-arbitrage are violated by the above premiums. *True or false?*

Problem 4.10. (5 points)
The current price of a share of stock **S** is \$100. The stock is assumed to be paying a continuous dividend with the dividend yield of 0.04.
Assume that the continuously compounded interest rate equals 0.05
Consider the following European gap options with the same exercise date in one year and the same underlying asset S.

I Gap call with strike price 100 and trigger price 100
II Gap put with strike price 100 and trigger price 100
III Gap call with strike price 100 and trigger price 110
IV Gap call with strike price 110 and trigger price 100
V Gap call with strike price 100 and trigger price 80.

Which one of the above options has the **highest** price?

4.3. **Bull and bear spreads.** Provide your final answer only to the following problem(s):

Problem 4.11. (2 points) A bear spread is a long position with respect to the underlying asset. *True or false?*

Problem 4.12. (5 points) The following three nine-month European put options are available in the market:

- a $120-strike put with the premium of $12,
- a $127-strike put with the premium of $10,
- a $130-strike put with the premium of $14.

Which of the following positions certainly exploits the arbitrage opportunity caused by the above put premia?

(a) Put bull spread.
(b) Put bear spread.
(c) Both of the above positions.
(d) There is no arbitrage opportunity.
(e) None of the above.

Please provide your **complete solutions** to the following problems.

Problem 4.13. (5 points) **Bear spreads in hedging**
An investor purchases a call option with an exercise price of $55 for $2.60. The same investor sells a call on the same asset with an exercise price of $60 for $1.40. At expiration, 3 months later, the asset price is $56.75. All other things being equal and given a continuously compouned annual interest rate of 4.0%, what is the profit to the investor?

Problem 4.14. (5 points) An investor acquires a call bull spread consisting of the call with strike $K_1 = 100$ and $K_2 = 110$ and with expiration in one year. The initial price of the $100-$strike call option equals $11.34, while the price of the $110-$strike option equals $7.74. At expiration, it turns out that the stock price equals $105. Given a continuously compouned annual interest rate of 5.0%, what is the profit to the investor?

(a) $3.78 loss
(b) $1.22 loss
(c) $1.22 gain
(d) $5 gain
(e) None of the above.

Problem 4.15. Consider a non-dividend-paying stock. Which of the following portfolios has the same payoff as a $(40, 50)-$bull spread?

(a) A **long** $(40, 50)-$collar and a short stock.
(b) A **short** $(40, 50)-$collar and a long stock.
(c) A **long** $40-$strike call, a **written** $50-$strike put, and a long stock.
(d) A **long** $40-$strike call, a **written** $50-$strike put, and a short stock.
(e) None of the above.