

UNIVERSITY OF TEXAS AT AUSTIN

HW Assignment 1

Outright purchase. Fully-leveraged portfolios. Long/short positions.

## 1.1. Fully-leveraged portfolios.

**Problem 1.1.** (5 points) Write down the definition of a *fully-leveraged portfolio*.

## 1.2. Long/short positions.

**Problem 1.2.** (5 points) Complete the following definition:A financial portfolio is said to be long with respect to an underlying asset if**Problem 1.3.** (5 points) Complete the following definition:A financial portfolio is said to be short with respect to an underlying asset if**Problem 1.4. "Partially-leveraged" purchase.**

Consider a continuous-dividend-paying stock with the dividend yield  $\delta$  whose market price at any time  $t$  is denoted by  $S(t)$ . You decide to purchase one share of this stock at time  $t=0$  and you partially finance your purchase by borrowing a portion  $\varphi$  of the initial stock price at the continuously-compounded, risk-free interest rate  $r$  to be repaid in full at time  $t=T$ .

- (i) (2 points) What is the initial cost of your portfolio?
- (ii) (3 points) What is the payoff of your portfolio?
- (iii) (3 points) What is the profit of your portfolio?
- (iv) (2 points) How does the profit curve of your portfolio compare to the profit curve of an outright purchase of the same asset? How about the fully-leveraged purchase of the same asset?

**Problem 1.5.** (3 points) Consider an outright purchase of a share of continuous-dividend-paying stock whose current price is \$80 per share and whose dividend yield is 0.02. Let the continuously compounded, risk-free interest rate be equal to 0.04. What is the time  $t=2$  break-even stock price for this investment?**Problem 1.6.** (3 points) Bertram sells short 10 shares of a continuous-dividend-paying stock. The time  $t=0$  price of this stock is \$100 and its dividend yield is 0.03. Assume that the continuously compounded, risk-free interest rate equals 0.06. If Bertram closes the short sale in six months, what is his break-even final stock price?**Problem 1.7.** The market in which Inaho trades has three possibilities for investment:

- a risk-free asset with the continuously compounded, risk-free interest rate equal to  $r$ ;
- a risky asset whose price is denoted by  $S(t), t \geq 0$  and whose dividend yield is  $\delta_S$ ;
- a risky asset whose price is denoted by  $Q(t), t \geq 0$  and whose dividend yield is  $\delta_Q$ .

Initially, the market prices of assets  $S$  and  $Q$  are equal. Inaho opens a one-share short position in the asset  $S$  and uses the proceeds of the short sale to purchase a share of the asset  $Q$ . At time  $t=T$ , Inaho sells the shares of asset  $Q$  she owns and closes the short sale of the asset  $S$ .

- (i) (2 points) What is the initial cost of this portfolio?
- (ii) (5 points) What is the profit of this portfolio?
- (iii) (3 points) What is the condition on the ratio of the final prices of assets  $S$  and  $Q$  for Inaho to break even?

**Problem 1.8.** (9 points) Let the current price of a continuous-dividend-paying stock be \$40 and let its dividend yield be equal to 0.01. The continuously compounded, risk-free interest rate is 0.04. You model the distribution of the time-1 price of the above stock as follows:

$$S(1) \sim \begin{cases} 45, & \text{with probability } 1/4, \\ 42, & \text{with probability } 1/2, \\ 38, & \text{with probability } 1/4. \end{cases}$$

What is your expected profit under the above model, if you invest in one share of stock at time-0 and liquidate your investment at time-1?