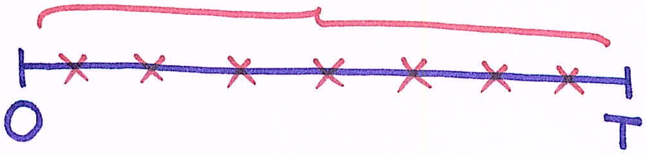
01/26/2018.

Review: STATIC PORTFOLIOS

NO INTERMEDIATE CASHFLOWS!



Initial Cost   Payoff

$$\boxed{\text{PROFIT} := \text{PAYOFF} - FV_{0,T}(\text{INITIAL COST})}$$

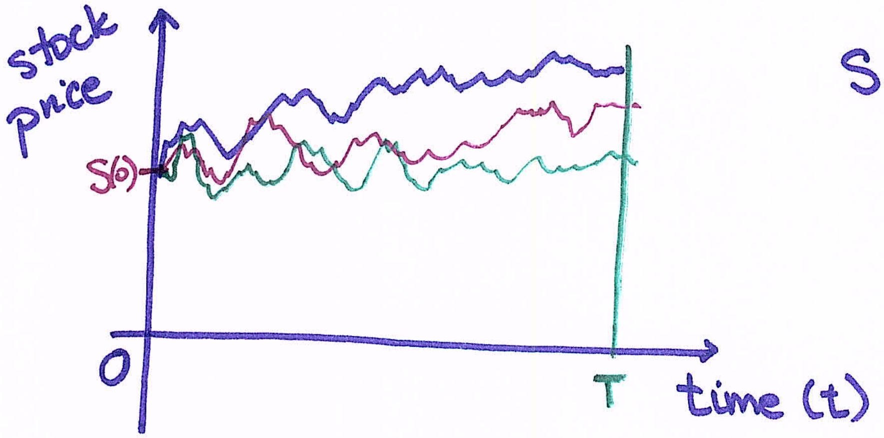
Example. TAKING A SIMPLE LOAN.

L … the loan amount
T … the loan term
r … continuously compounded, risk-free i.r.

$\Rightarrow$ Initial Cost : $-L$
Payoff : $-Le^{rT}$

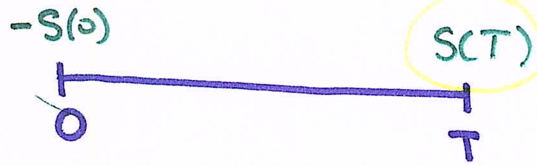
$\Rightarrow$ Profit $= -Le^{rT} + FV_{0,T}(+L)$
$= -Le^{rT} + Le^{rT} = 0$ ∎

Example. THE OUTRIGHT PURCHASE OF A NON-DIVIDEND-PAYING STOCK



$S(T)$ ... final asset price is a random variable

the timeline:



the cashflows

$\Rightarrow$ Initial Cost: $S(0)$

Payoff: $S(T)$ random variable

$\boxed{\text{Introduce:}}$ s ... an independent argument

... stands for the FINAL ASSET PRICE

... "placeholder" for the rnd variable $S(T)$

We can define the

payoff function

which describes the dependence of the investor's payoff on $s \longleftrightarrow S(T)$.

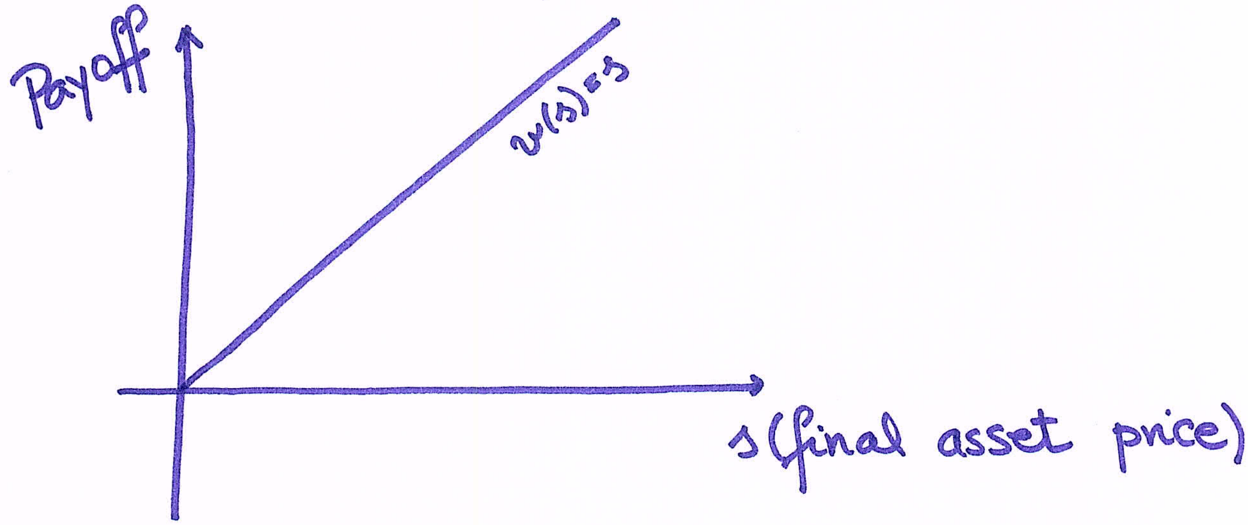
Notation: v ... payoff function

$\Rightarrow v(s)$ ... the investor's payoff if the final asset price equals s

⇒ In the present example:

<u>The Payoff Function</u> : $v(s) = s$ $\boxed{\text{the identity !}}$

When we graph the payoff function, we get the PAYOFF DIAGRAM/CURVE:



$$\begin{aligned} \text{Profit} &= \text{Payoff} - FV_{0,T}(\text{Initial Cost}) \\ &= S(T) - FV_{0,T}(S(0)) \\ &= S(T) - \underbrace{S(0)e^{rT}}_{\text{constant}} \end{aligned}$$

r... cont. comp., risk-free i.r.

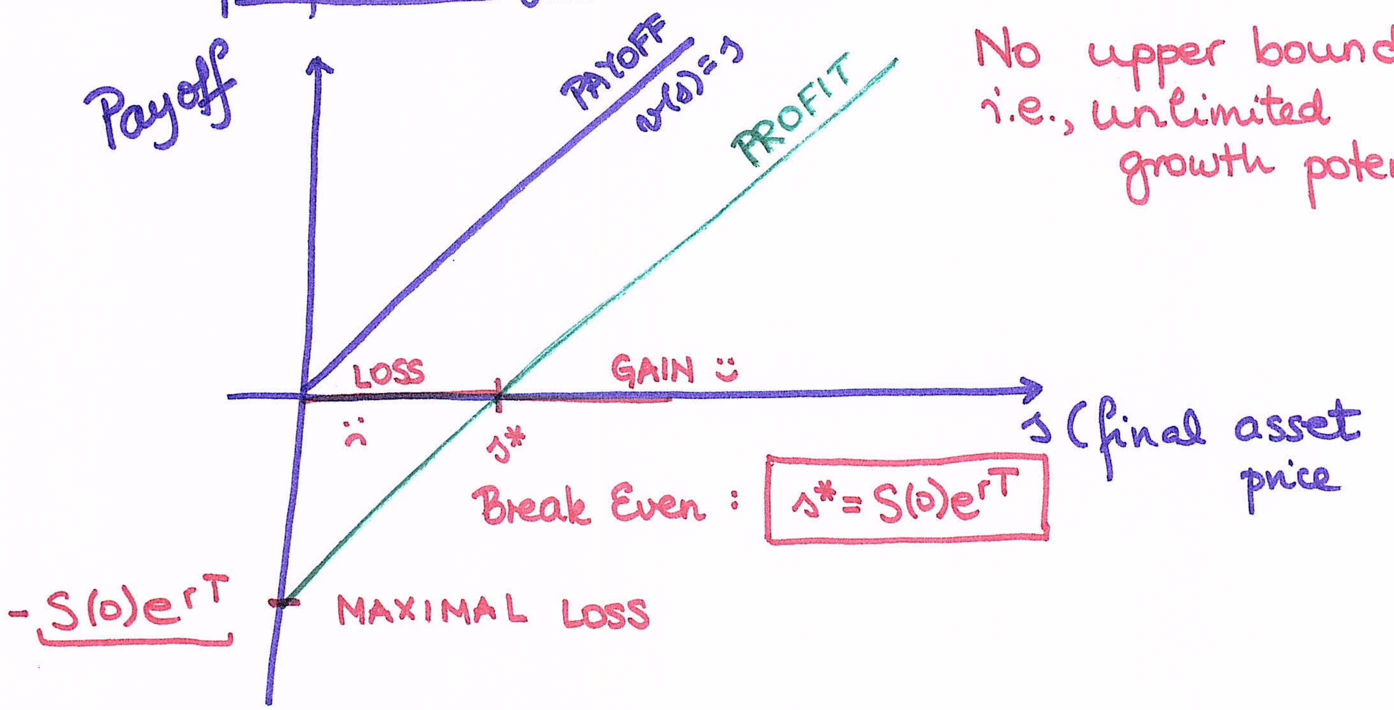
Introduce the <u>profit function</u>:

$$\boxed{\underbrace{v(s)}_{\text{Payoff}} - FV_{0,T}(\text{Init. Cost})}$$

⇒ In this example, the profit function:

$$\boxed{s - S(0)e^{rT}}$$

⇒ The profit diagram / curve:



No upper bound, i.e., unlimited growth potential.

IF PAYOFF/PROFIT is increasing (not necessarily strictly) as a function of the final asset price (s), we say the portfolio is

long with respect to the underlying (asset).

Example. OUTRIGHT PURCHASE OF ONE SHARE OF A CONTINUOUS-DIVIDEND-PAYING STOCK W/ DIVIDEND YIELD δ

→ : Initial Cost : $S(0)$

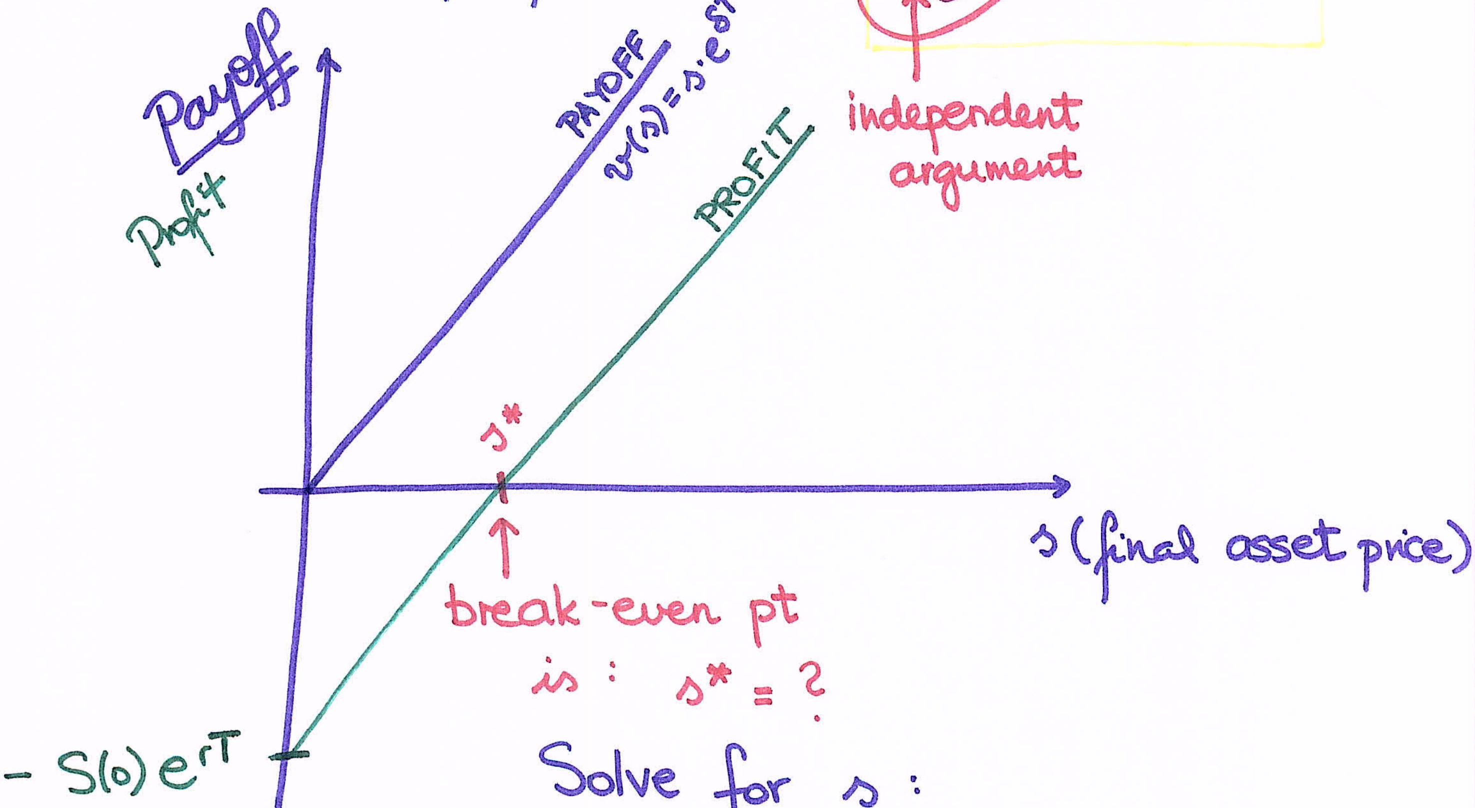
Payoff : $\underbrace{e^{\delta T}}_{\text{the \# of shares owned @ time-T}} \cdot S(T)$

⇒ The payoff function is

$$\boxed{v(s) = s \cdot e^{\delta T}}$$

$\Rightarrow$ <u>Profit</u>: $S(T)\cdot e^{\delta T} - S(0)e^{rT}$

$\Rightarrow$ The <u>profit function</u> is $\boxed{\underbrace{s}_{\text{independent argument}}\overbrace{e^{\delta T}}^{\text{a constant}} - S(0)e^{rT}}$



Solve for s:

$$s e^{\delta T} - S(0)e^{rT} = 0$$

$$\Rightarrow \quad s e^{\delta T} = S(0)e^{rT}$$

$$\Rightarrow \quad \boxed{s^* = S(0)\,e^{(r-\delta)T}}$$

Try to remember this expression!