UNIVERSITY OF TEXAS AT AUSTIN

Lecture 15

Convexity of call/put prices. Butterfly spreads.

15.1. Convex functions.

Definition 15.1. A function $f : [0, \infty) \to \mathbb{R}$ is said to be *convex* if for every $x_L < x_R$ and every $0 \le \lambda \le 1$ we have

$$f(\lambda x_L + (1-\lambda)x_R) \le \lambda f(x_L) + (1-\lambda) f(x_R)$$

15.2. Convexity of call-option prices.

Theorem 15.2. *European call-option prices are* **convex** *as functions of the strike price.*

Problem 15.1. Formally express the convexity property of the call-option prices in terms of the three ordered strikes $K_1 < K_2 < K_3$.

Solution: We have that

$$V_C(K_2) \le \lambda V_C(K_1) + (1-\lambda) V_C(K_3)$$

with $\lambda = \frac{K_3 - K_2}{K_3 - K_1}$.

Example 15.3. Assume, to the contrary, that there exist $K_1 < K_2 < K_3$ such that

$$V_C(K_2) > \lambda V_C(K_1) + (1-\lambda) V_C(K_3)$$

with $\lambda = \frac{K_3 - K_2}{K_3 - K_1}$. Construct an arbitrage portfolio and show that your proposed portfolio is, indeed, an arbitrage portfolio.

Portfolio construction. We propose the following portfolio:

- ______________ $\boxed{}$ K_1-strike call(s),
- ______________ $\boxed{}$ K_2-strike call(s), and
- ______________ $\boxed{}$ K_3-strike call(s).

Verification. The initial cost of the above portfolio equals:
Solution:

$$\lambda V_C(K_1) + (1-\lambda) V_C(K_3) - V_C(K_2) < 0$$

As usual, it is easiest to study the payoff by drawing the payoff function.