

UNIVERSITY OF TEXAS AT AUSTIN

Lecture 4

European call options

4.1. **The definition.** The mechanics of the *European call option* are the following:

- at time -0 : (1) the contract is agreed upon between the buyer and the writer of the option,
 (2) the logistics of the contract are worked out (these include the underlying asset, the *expiration date* T and the *strike price* K),
 (3) the buyer of the option pays a *premium* to the writer of the call option;
 at time $-T$: the buyer of the option can **choose** whether he/she will purchase the underlying asset for the strike price K .

4.1.1. *The buyer's perspective.* The buyer of the European call option is the party who is entitled from the “**optionality**” in the term itself. For the buyer, the call is a **non-binding** agreement in the sense that he/she has the **right, but not the obligation** to buy the underlying asset on the expiration date. This feature is preserved in the overwhelming majority of derivative securities with the word “option” in their names.

We will consider all of the characters involved in our trades to be **rational** in the sense that they will always take the course of action which benefits them the most financially speaking. So, let us figure out what the rational buyer would do at time $-T$ – whether he/she would purchase the asset or not. The buyer's behavior will be dictated by the price of the underlying asset at time $-T$.

If the final asset price exceeds the strike price, i.e., if $S(T) > K$, this means that the buyer is able to purchase the asset “cheaply”. In our perfectly liquid markets, he/she would be able to purchase the asset for K and immediately sell it for the market price of $S(T)$. The effect of these actions on his/her pocketbook is the payoff of $S(T) - K$.

To the contrary, if $S(T) \leq K$, the buyer would be better off purchasing the asset at the market price. So, he/she will simply walk away from the contract incurring the payoff of 0.

Combining the above two scenarios, we get the following expression for the buyer's payoff:

$$V_C(T) = \begin{cases} S(T) - K, & \text{if } S(T) \geq K \\ 0, & \text{if } S(T) < K. \end{cases}$$

After a (very short) while, it gets tedious to repeat the above expression, so we write

$$V(T) = \max(S(T) - K, 0) = (S(T) - K) \vee 0.$$

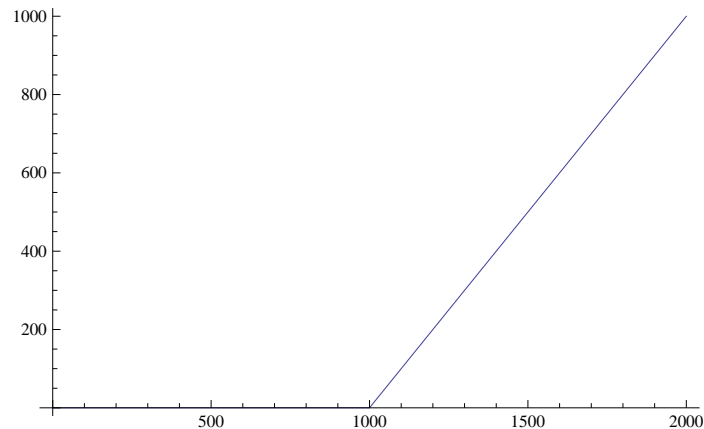
To make the notation even more compact we introduce the following shorthand

$$(x)_+ = x \vee 0.$$

With this notation, we can finally write the buyer's payoff as

$$V(T) = (S(T) - K)_+.$$

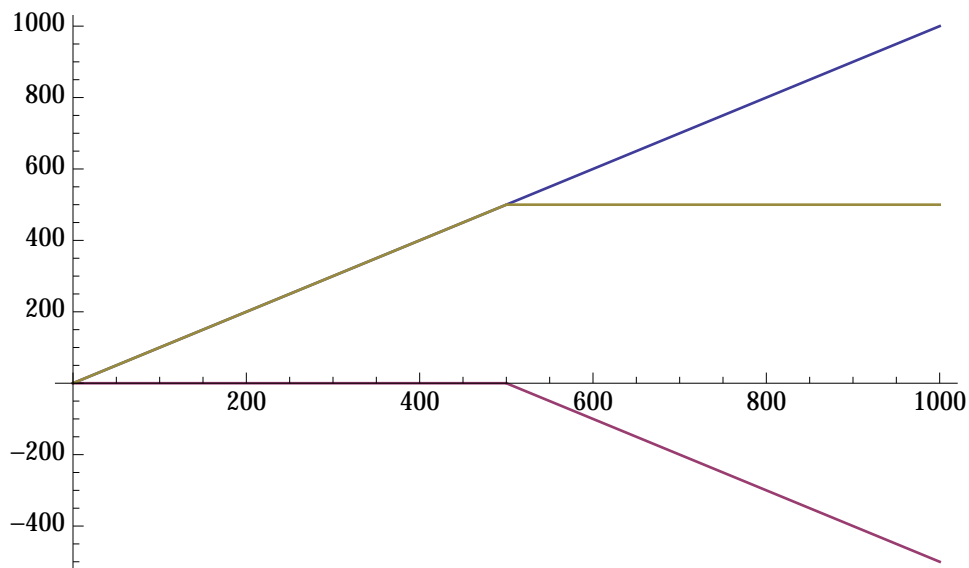
For $K = 1000$, we get the following payoff curve:



Looking at the graph above, we see that the call option's payoff is increasing in the asset price having an infinite upward potential. On the other hand, there is no downside to the buyer for final asset prices lower than the strike. We should not forget that the buyer is supposed to pay the premium at $t = 0$. This will affect the profit curve. Also,

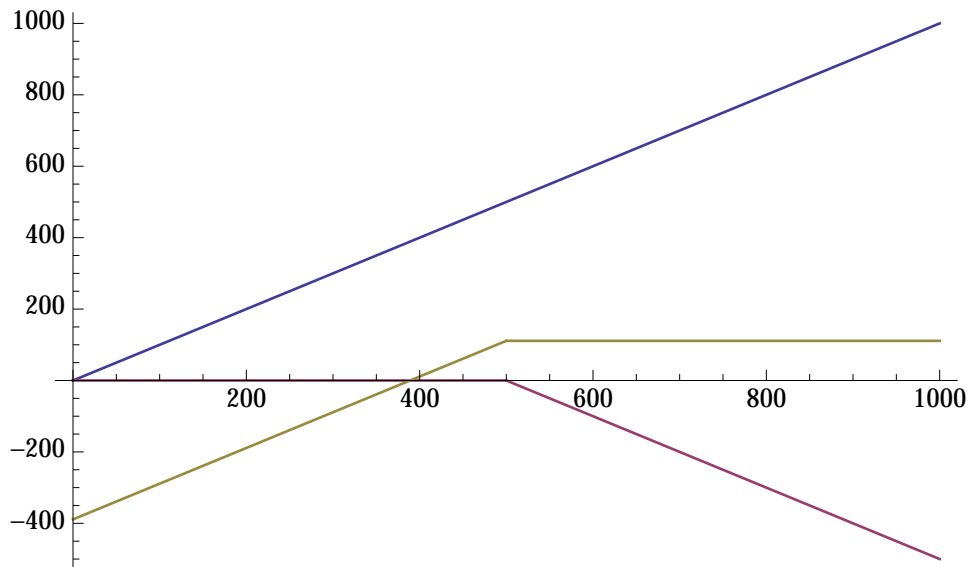
4.2. Hedging using European calls.

4.2.1. *The producer's perspective.* Recall that a producer of goods has an inherent long position in that asset. The long/bought call option is itself a long position with respect to the underlying. So, to offset his/her inherent long position, the producer should short/write the call option. The payoff curve of the hedged portfolio is shown in the graph below.



The blue line is the **unhedged** payoff. The red line is the **hedge** – the **short** European call (with the strike price equal to 500). The green line is the payoff curve of the **hedged portfolio**, i.e., the sum of the payoffs of the unhedged position and the short European call. At first sight, we might believe that the producer would be better off if (s)he had decided to hedge by selling forward. But, let us look at the profit curve in this case. Assume that the total fixed costs of production accumulate to 450, and that the call's price equals 10. With

the continuously compounded risk-free interest rate equal to 0.05 and expiration date in a year, we get the profit curve in the following figure:



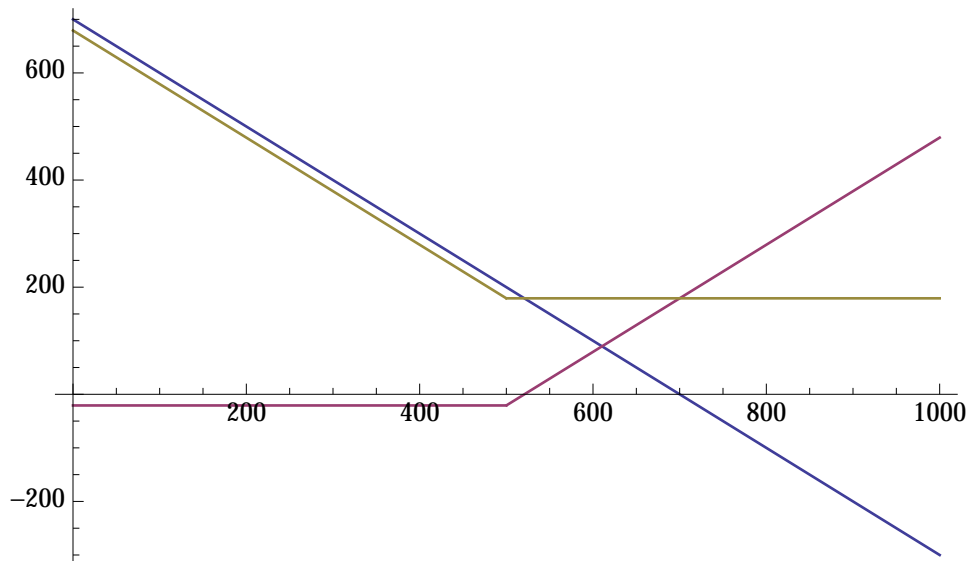
A closer look at the curve above reveals that for final asset prices exceeding the call's strike, the profit exceeds the profit in the case of the short-forward hedge with the forward price equal to the strike price. As the strike price is shifted, the “amount of insurance” is adjusted as well. This shift also reflected in the change in the call's price. The mechanism is similar to the effect the dividend has on the insurance-policy premium in classical insurance. We will look at both the loose analogy between financial hedging and classical insurance, and the monotonicity of European call prices with respect to the changes of the strike price.

Example 4.1. Sweet potato call options with the exercise date in six months (at harvest time) and a \$15.00 strike price (per carton) are trading for a \$1.50 premium. Farmer Brown decides to hedge his 10,000 cartons of sweet potatoes buy writing 10,000 of the above call options. The total fixed costs of producing his entire sweet-potato crop is \$120,000. Assume the continuously compounded risk-free interest rate equal to 0.04. What is farmer Brown's profit if the market price of sweet potatoes turns out to be \$14.50 per carton at harvest time?

Solution:

$$10000(14.50 + 1.50e^{0.04/2}) - 120000 = 40303$$

4.2.2. *The buyer's perspective.* The buyer's perspective is symmetric to the producer's perspective. The buyer has an inherent short position in the “input” good (s)he needs to purchase. The sensible course of action would be for the buyer to buy a European call on this good to hedge against this particular price risk. The following graph shows the profit of the (total) hedged position of our commodity buyer.



Can you identify which curve in the figure above represents which aspect of the hedged portfolio?

Example 4.2. The “Babkas, Brownies and Beyond” bakery sells blueberry muffins for \$3.00 per muffin. The bakery will need to buy 100 lbs of blueberries in six months to produce the 1600 muffins needed for the “Greater Springfield Blueberry Jamboree”. Non-blueberry costs total \$2,500. Assume that the continuously compounded risk-free interest rate equals 0.04. Local farmers are financially sophisticated. Our bakery uses one hundred \$1.60-strike, six-month call options (each on a pound of blueberries) to hedge against rising prices of blueberries. The calls can be bought for \$0.15 per call. Assume that the market price of a pound of blueberries is \$1.65 in six months. What is the profit of the bakery’s hedged portfolio?

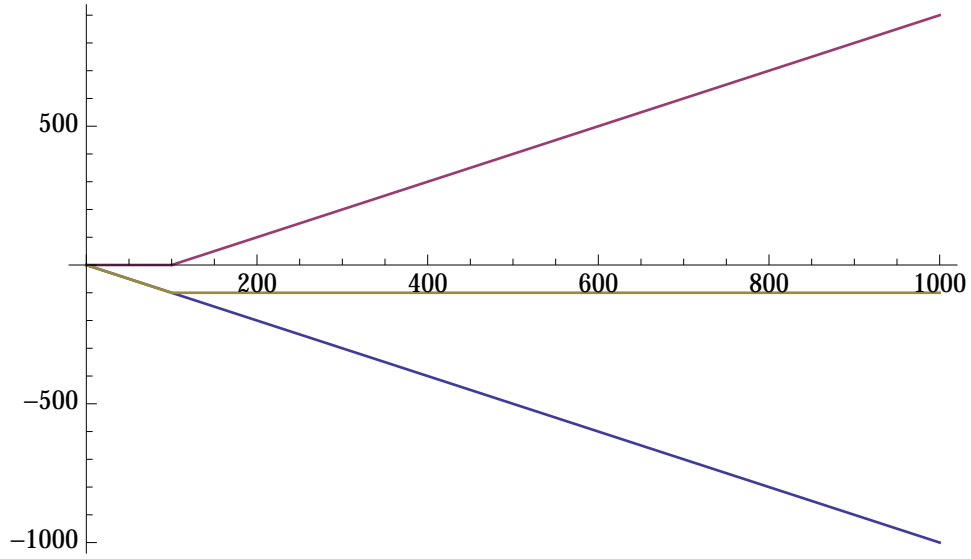
Solution:

$$1600 \cdot 3 - 2500 - 100 \cdot 1.65 + 100(1.65 - 1.60)_+ - 100 \cdot 0.15e^{0.04/2} = 2124.70.$$

4.2.3. *The short seller’s perspective.* The stock short seller has a short position in the asset and would use a long call to insure this position. Let us look at the following situation:

- the initial stock price is $S(0) = 100$; the short seller receives the proceeds of the short sale at time -0 ;
- the continuously compounded risk-free interest rate equals 0.04;
- the short sale is closed at time $T = 1$;
- a 100-strike, one-year European call on the stock is sold for \$5 at time -0 .

What is the short seller’s payoff curve?



What about the profit curve?

