

UNIVERSITY OF TEXAS AT AUSTIN

Lecture 4

Payoff and profit – the simplest scenario

4.1. **The time-line.** Before we look into specific derivatives, let us introduce two concepts of interest: the *payoff* and the *profit*. The first part of the course focuses almost exclusively on the study of the payoff and the profit structures of a small set of specific investment portfolios (collections of assets and financial instruments that an investor is trading in during our time frame $[0, T]$).

Let us start with a simple time-line. We choose the beginning of time, i.e., time -0 , so that it suits our purposes. Most often, this is the time when the first investment of interest happens, e.g., a purchase of a number of shares takes place or a loan is taken out. We frequently refer to time -0 as “today”. The simplest scenario is the one in which the investor makes no trades until time $-T$. Likewise, there are no other cashflows until time $-T$. For instance, there are no discrete dividends and continuous dividends are all automatically and continuously reinvested in the same stock. If this is the case, it is fairly simple for the investor to assess all the cashflows associated with the position.

4.2. **The initial cost.** At time -0 , the cashflow associated with our portfolio is usually referred to as the *initial cost*. The term might be misleading; while it is usual that the initial cashflow is an investment on behalf of the agent, it is also possible for the agent to initially receive an amount of money.

- (1) If the initial cost is **positive**, it means that the investor **gave up** that particular amount.
- (2) If the initial cost is a **negative** value, this signifies the fact that the investor initially **received** the absolute value of the initial cost.
- (3) If the initial cost is **zero**, it means that there were no cashflows at time -0 ; if this is the case, we refer to the portfolio in question as *fully leveraged*.

Example 4.1. An outright purchase Let the agent purchase 100 shares of non-dividend-paying stock valued at \$50 per share at time -0 . Then, the agent’s initial cost equals $100 \times 50 = 5,000$.

Example 4.2. Short sale Assume that an agent short sells one share of the stock with the spot price of \$500 per share at time -0 . Then, the agent’s initial cost needs to reflect the fact the (s)he initially receives the proceeds of the short sale. It equals -500 .

Example 4.3. A fully leveraged purchase Assume that an investor wishes to purchase 1000 oz of gold at a price of \$125.00 per ounce. (S)he does not have the funds for this purchase, so decides to borrow the required money. His/her initial cashflow is, therefore,

$$1000 \times 125 - 1000 \times 125 = 0.$$

The initial cost of this portfolio is zero.

While our terminology might seem at odds with the everyday usage of the term “cost” to always signify an outflow of cash, you will soon realize that this is a useful blanket term.

4.3. **The payoff.** The time- T net cashflow is usually referred to as the *payoff* of the portfolio. Again, the sign of the payoff reflects whether the investor receives the funds or not.

- (1) If the payoff is **positive**, it means that the investor **receives** that particular amount.
- (2) If the payoff is a **negative** value, this signifies the fact that the investor has to **pay** the absolute value of the payoff.

Example 4.4. An outright purchase (cont'd) Recall the above agent who purchased 100 shares of non-dividend-paying stock time-0. This agent decides to liquidate this investment at time- T . If the market price of stock is \$80 per share at that time, the investor's payoff is $80 \times 100 = 8,000$.

Example 4.5. Short sale (cont'd) Reconsider the agent who sold short one share of stock time-0. In addition, assume that this stock pays continuous dividends with the yield 0.02. We ignore margin requirements. At time-1, the investor decides to close the short sale. The observed market price of the stock is \$500 at time-1. The short-seller needs to return $e^{0.02}$ shares, so that (s)he needs to spend $500e^{0.02} = 510.10$. The payoff of this portfolio is -510.10 .

Example 4.6. A fully leveraged purchase (cont'd) Remember the an investor who purchased 1000 oz of gold using borrowed funds amounting to 125,000. Let the continuously-compounded risk-free interest rate equal 0.04. The investor decides to sell the gold at time-1 and repay the loan. Let the time-1 price of gold be \$120 per oz. Then, the payoff equals

$$120,000 - 125,000e^{0.04} = -10101.35.$$

The negative payoff indicates that the investor needs to contribute his/her own funds.

4.4. **The profit.** Looking solely at the payoff does not reflect the "quality" of an investment portfolio fully. We prefer to look at how the portfolio compares to a simple investment at the risk-free rate. The concept reflecting this point of view is that of the *profit*. In the simplest case we currently focus on, we define the profit as the difference between the payoff and the **future value** at time- T of the initial cost.

Example 4.7. An outright purchase (cont'd) Once more, recall the agent who purchased 100 shares of non-dividend-paying stock time-0. Let us say that this agent decided to sell the shares at time-1 at the market price of \$80 per share. Assume that the continuously-compounded risk-free interest rate equals 0.05. This portfolio's profit is

$$8000 - 5000e^{0.05} = 2743.64.$$

Example 4.8. Short sale (cont'd) Again, reconsider the short-seller above, assuming the continuously-compounded risk-free interest rate of 0.05. The profit is

$$-510.10 - (-500)e^{0.05} = 15.53.$$

The situation does not look as bleak as it did when we just focused on the payoff.

Remark 4.9. (1) In the case of a fully-leveraged portfolio, the payoff and the profit are always equal.

- (2) If the profit is positive, we refer to it as the *gain*. If it is negative, we call it a *loss*.

4.5. **The payoff curve.** In our models, it is usually the case that we deal with exactly one risky asset as the source of uncertainty in our portfolios. This means that the payoff and profit structures of our investment portfolios are known at time $t=0$ up to the uncertainty regarding the asset price at the end of our time-horizon. To understand the possible behavior of the portfolio and its financial consequences, it helps to consider what the payoff and profit would be for all possible prices of the risky asset at time $t=T$. We do not know in advance what this price is going to be, and in the absence of a specific model, we have to consider all nonnegative values as possible final stock prices. The mathematical formalism which allows us to do this is to look at the payoff as a function of an argument s where s stands for the possible final asset prices. It helps to graph this function thus obtaining the *payoff curve*. The x -axis corresponds to the payoff function's argument s (still, the “placeholder” for the possible values of $S(T)$).