University of Texas at Austin

Lecture 7

An introduction to European put options. Moneyness.

7.1. **Put options.** A *put option* gives the owner the **right** – but **not** the obligation – to sell the underlying asset at a predetermined price during a predetermined time period. The seller of a put option is **obligated** to buy if asked. The mechanics of the European put option are the following:

at time−0: (1) the contract is agreed upon between the buyer and the writer of the option,
(2) the logistics of the contract are worked out (these include the underlying asset, the *expiration date* T and the *strike/exercise price* K),
(3) the buyer of the option pays a *premium* to the writer;

at time−T: the buyer of the option can **choose** whether he/she will sell the underlying asset for the strike price K. The writer of the option is bound to fulfill the put-option buyer's choice.

7.1.1. *The European put option payoff.* We already went through a similar procedure with European call options, so we will just briefly repeat the mental exercise and figure out the European-put-buyer's profit.

If the strike price K exceeds the final asset price $S(T)$, i.e., if $S(T) < K$, this means that the put-option holder is able to sell the asset for a higher price than he/she would be able to in the market. In fact, in our perfectly liquid markets, he/she would be able to purchase the asset for $S(T)$ and immediately sell it to the put writer for the strike price K. The payoff is, thus, $S(T) - K$.

To the contrary, if $S(T) \geq K$, the put-option owner would be better of selling the asset at the market price. So, he/she will simply walk away from the contract incurring the payoff of 0.

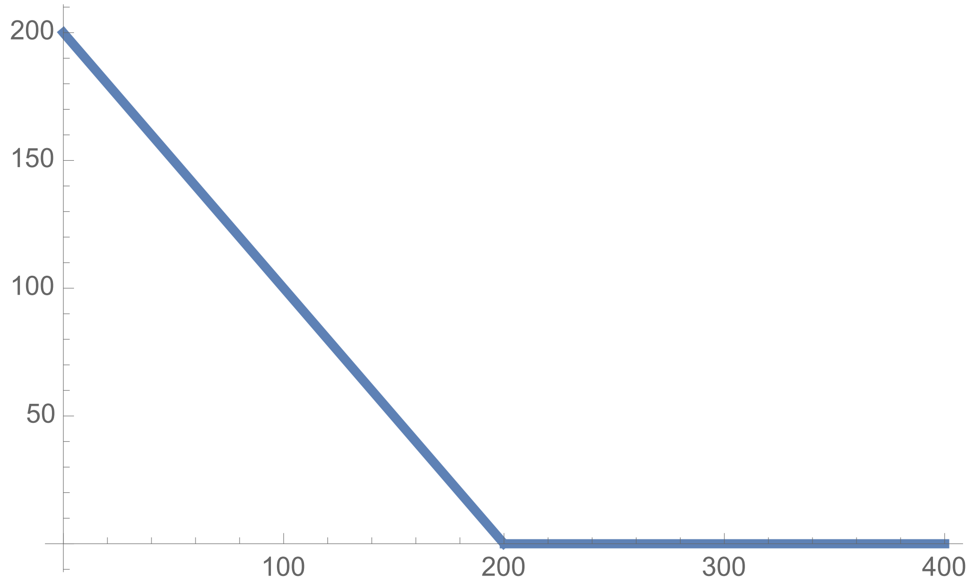
Combining the above two states of the world, we get the following expression for the long-put-option payoff:

$$V_P(T) = \max(K - S(T), 0) = (K - S(T))_+.$$

So, the payoff function for a put option is

$$v_P(s) = (K - s)_+.$$

For $K = 1000$, we get the payoff curve below (in blue). The buyer is supposed to pay the premium at $t = 0$. This will affect the profit curve. For instance, if the intial premium for this option with exercise date in one year equals \$50 and if the continusouly compounded interest rate equals $r = 0.06$, then the profit curve is the one graphed below in red.



Looking at the graph above, we see that the put-option payoff (as well as profit) is decreasing in the asset price and bounded from above by the strike price K.

Example 7.1. Put option on a market index
Consider a put option on a market index with exercise date in six months and with strike price $K = 1000$. Assume that the premium for this option equals $V_P(0) = \$80$ and that the effective interest rate for the six-month period equals $i = 0.03$. The payoff function is

$$v_P(s) = (1000 - s)_+$$

and the profit fuction is

$$v_P(s) - V_P(0)(1+i) = (1000 - s)_+ - 80 \cdot 1.03 = (1000 - s)_+ - 82.4.$$

Once the exercise date is reached, one gets to observe the final index value and calculate the realized payoff and profit. For instance:

(1) If the final index value equals $S(T) = 1,050$, the put-owner's payoff is 0 (the option is not even exercised). The profit is, hence, -82.40. So, the owner of the option experiences a **loss** of \$82.40.
(2) If the final index value equals $S(T) = 800$, the payoff is $1000 - 800 = 200$ (the option is, indeed, exercised). The profit is, hence, $200 - 82.40 = 117.60$. So, the owner of the option **gains** of \$117.60.

Remark 7.2. Two positions in the market with the the payoff of one being the exact negative payoff of the other are said to be **opposites** of each other. In particular,

- a **purchased** call is the opposite of a **written** call;
- a **purchased** put option is the opposite of a **written** put.

7.1.2. *Problems.*

Problem 7.1. The initial price of the market index is \$900. After 3 months the market index is priced at \$915. The nominal rate of interest convertible monthly is 4.8%. The premium on the put, with a strike price of \$930, is \$8.00. What is the profit at expiration for a **long** put?

(a) \$15.00 loss
(b) \$6.90 loss
(c) \$6.90 gain
(d) \$15.00 gain
(e) None of the above.

Solution: (c)
The profit from a position is defined as the position's payoff minus the future value of the initial cost.
If $S(T) = 915$ denotes the price of the market index at time $T = 0.25$ (i.e., in three months), then the payoff of the long put is $(K - S(T))_+$, where $K = 930$ denotes the strike of the put. So, since $K > S(T)$, the payoff is

$$(930 - 915)_+ = 15.$$

The future value of the initial put premium is

$$8(1 + 0.004)^3 = 8.0964.$$

So, the profit is

$$15 - 8.0964 = 6.90.$$

Problem 7.2. Sample FM(DM) #12
Consider a European put option on a stock index without dividends, with 6 months to expiration, and a strike price of 1,000. Suppose that the nominal annual risk-free rate is 4% **convertible semiannually**, and that the put costs 74.20 now. What price must the index be in 6 months so that being long the put would produce the same profit as being short the put?

A. 922.83
B. 924.32
C. 1,000.00
D. 1,075.68
E. 1,077.17

Solution: (b)
Method I. A quick and insightful way of solving this problem is by realizing that the long-put and the short-put profits are negatives of each other. So, the only way they can be equal is at the "break-even" point. We solve for s in

$$(K - s)_+ - V_P(0)\left(1 + \frac{i^{(2)}}{2}\right) = (1000 - s)_+ - 74.20(1.02) = 0.$$

The solution is $s = 924.32$.
Method II. This is the more pedestrian method. The long-put profit is

$$(K - s)_+ - V_P(0)\left(1 + \frac{i^{(2)}}{2}\right) = (1000 - s)_+ - 74.20(1.02).$$

The short-put profit is the exact negative of the expression above, i.e.,

$$-(K-s)_+ + V_P(0)\left(1+\frac{i^{(2)}}{2}\right) = -(1000-s)_+ + 74.20(1.02).$$

So, algebraically, we need to solve for s in the equation

$$(1000-s)_+ - 74.20(1.02) = -(1000-s)_+ + 74.20(1.02) \quad \Leftrightarrow \quad 2(1000-s)_+ = 2\cdot 74.20(1.02)$$
$$\Leftrightarrow \quad (1000-s)_+ = 74.20(1.02).$$

We get the same answer as above, of course.

Problem 7.3. Farmer Shaun is producing sweet potatoes. He intends to harvest $10,000-$cartons' worth in six months. His total costs are \$12.00 per carton.

He wishes to hedge using European put options. There are two puts on sweet potatoes with the exercise date in six months available: one with the strike price of \$13 per carton and another with the strike price of \$15 per carton. Their premiums are \$0.15 and \$0.18, respectvely.

Assume that the prevailing risk-free interest rate is 4% effective for the half-year period.

At harvest time, in six months, it turns out that the sweet-potato spot price equals \$14. What would Farmer Shaun's profit be if he had decided to hedge using the \$13-strike put versus his profit if he had decided to use the \$15-strike put to hedge?

Solution: Farmer Shaun's unhedged position has the following profit:

$$10,000(S(T)-12)$$

where $S(T)$ stands for the spot price of sweet potatoes in six months.

If he decided to hedge using put options, he would **long** the put. So, the profit of the \$13-strike-put hedge would be:

$$10,000(13-S(T))_+ - 10,000\times 0.15\times 1.04.$$

The profit of the \$15-strike-put hedge would be:

$$10,000(15-S(T))_+ - 10,000\times 0.18\times 1.04.$$

The profit of the hedged position with the given $S(T)=14$ in the first case equals

$$10,000(14-12-0.15\times 1.04) = 18,440.$$

For the second insurance strategy, the profit is

$$10,000(14-12+(15-14)-0.18\times 1.04) = 28,128.$$

7.1.3. *Suggested problems.* McDonald: #2.3, #2.5, #2.14.

7.2. **Moneyness.** The **moneyness** of an option reflects whether an option would cause a positive, negative or zero cashflow were to be exercised **immediately**. More precisely, at any time $t \in [0, T]$, an option is said to be:

(1) *in-the-money* – the owner of the option would receive a **strictly positive** cashflow were the option exercised immediately;
(2) *at-the-money* – the owner of the option would receive a **zero** cashflow were the option exercised immediately;
(3) *out-of-the money* – the owner of the option would receive a **strictly negative** cashflow were the option exercised immediately.

Example 7.3. **Moneyness of a put option**
Consider a put option with strike $K = 100$. If the inital price $S(0)$ of the underlying asset equals:

(1) 95 – then the option is in-the-money;
(2) 100 – then the option is at-the-money;
(3) 105 – then the option is out-of-the-money.

Imagine that we are half-way through the life of the option, i.e., we have reached time $T/2$. We can observe the price of the underlying asset at that time too. We denote this value by $S(T/2)$, and we can also state that at time$-T/2$

(1) if $S(T/2) > 100$ the put option is out-of-the-money;
(2) if $S(T/2) = 100$ the put option is at-the-money;
(3) if $S(T/2) < 100$ the put option is in-the-money.