UNIVERSITY OF TEXAS AT AUSTIN

Lecture 6

European call options

6.1. **The definition.** The mechanics of the *European call option* are the following:

at time-0: (1) the contract is agreed upon between the buyer and the writer of the option,
(2) the logistics of the contract are worked out (these include the underlying asset, the *expiration date* T and the *strike/exercise price* K),
(3) the buyer of the option pays a *premium* to the writer of the call option;

at time$-T$: the buyer of the option can **choose** whether he/she will purchase the underlying asset for the strike price K.

6.1.1. *The buyer's perspective.* The buyer of the European call option is the party who is entitled to the **"optionallity"** in the term itself. For the buyer, the call is a **non-binding** agreement in the sense that he/she has the **right, but not the obligation** to buy the underlying asset on the expiration date. This feature is preserved in the overwhelming majority of derivative securities with the word "option" in their names.

We will consider all of the characters involved in our trades to be **rational** in the sense that they will always take the course of action which benefits them the most financially speaking. So, let us figure out what the rational buyer would do at time$-T$ – whether he/she would purchase the asset or not. The buyer's behavior will be dictated by the price of the underlying asset at time$-T$.

If the final asset price exceeds the strike price, i.e., if $S(T) > K$, this means that the buyer is able to purchase the asset "cheaply". In our perfectly liquid markets, he/she would be able to purchase the asset for K and immediately sell it for the market price of $S(T)$. The effect of these actions on his/her pocketbook is the payoff of $S(T) - K$.

To the contrary, if $S(T) \leq K$, the buyer would be better of purchasing the asset at the market price. So, he/she will simply walk away from the contract incurring the payoff of 0.

Combining the above two scenarios, we get the following expression for the buyer's payoff:

$$V_C(T) = \begin{cases} S(T) - K, & \text{if } S(T) \geq K \\ 0, & \text{if } S(T) < K. \end{cases}$$

After a (very short) while, it gets tedious to repeat the above expression, so we write

$$V(T) = \max(S(T) - K, 0) = (S(T) - K) \vee 0.$$

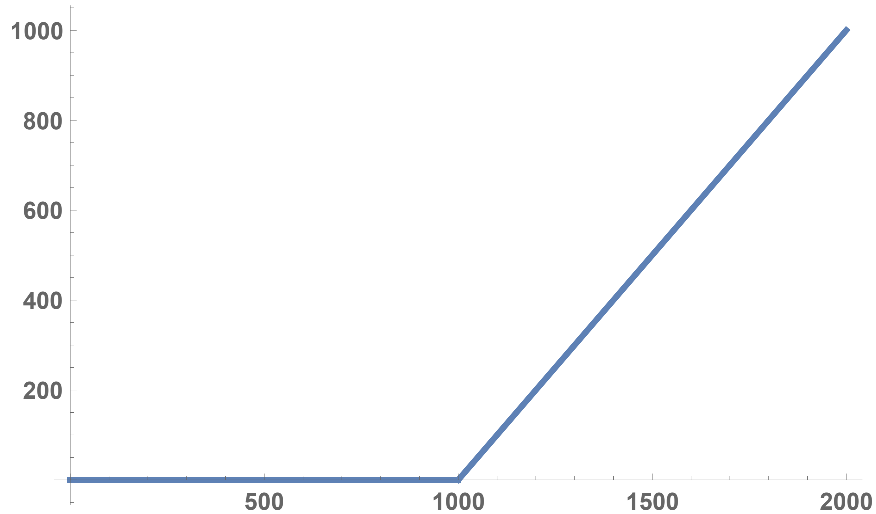
To make the notation even more compact we introduce the following shorthand

$$(x)_+ = x \vee 0.$$

With this notation, we can finally write the buyer's payoff as

$$V(T) = (S(T) - K)_+.$$

For $K = 1000$, we get the following payoff curve:



Looking at the graph above, we see that the call option's payoff is increasing in the asset price having an infinite upward potential. In other words, the call option is *long* with respect to the underlying asset. On the other hand, there is no downside to the buyer for final asset prices lower than the strike. We should not forget that the buyer is supposed to pay the premium at $t = 0$. This will affect the profit curve. Also,

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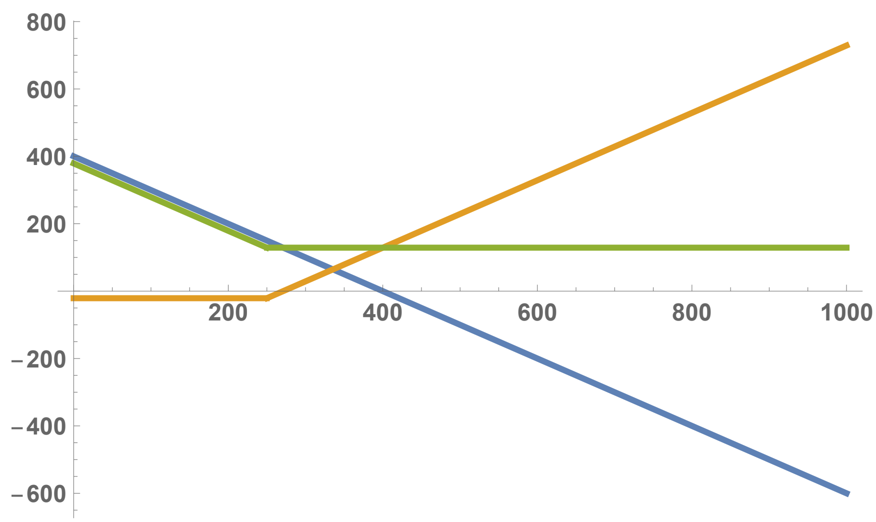
6.2. **Hedging using European calls.**

6.2.1. *The buyer's perspective.* The buyer has an inherent short position in the "input" good (s)he needs to purchase. The sensible course of action would be for the buyer to buy a European call on this good to hedge against this particular price risk. The following graph shows the profit of the (total) hedged position of our commodity buyer.



Can you identify which curve in the figure above represents which aspect of the hedged portfolio?

Example 6.1. The "Babkas, Brownies and Beyond" bakery sells blueberry muffins for \$3.00 per muffin. The bakery will need to buy 100 lbs of blueberries in six months to produce the 1600 muffins needed for the "Greater Springfield Blueberry Jamboree". Non-blueberry costs total \$2,500. Assume that the continuously compounded risk-free interest rate equals 0.04. Local farmers are financially sophisticated. Our bakery uses one hundred \$1.60-strike, six-month call options (each on a pound of blueberries) to hedge against rising prices of blueberries. The calls can be bought for \$0.15 per call. Assume that the market price of a pound of blueberries is \$1.65 in six months. What is the profit of the bakery's hedged portfolio?

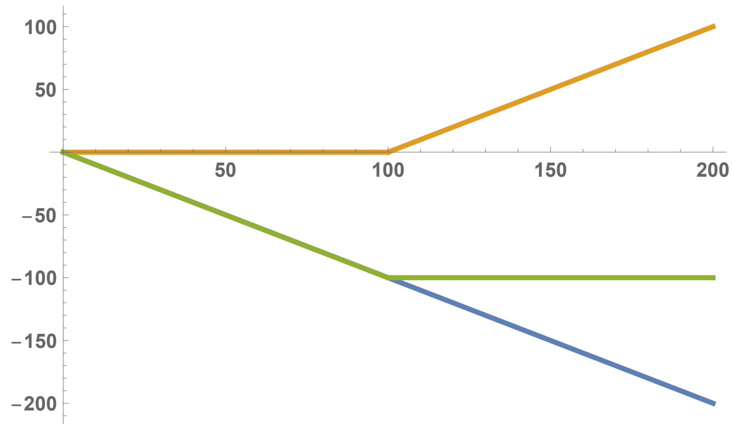
Solution:

$$1600 \cdot 3 - 2500 - 100 \cdot 1.65 + 100(1.65 - 1.60)_+ - 100 \cdot 0.15e^{0.04/2} = 2124.70.$$

6.2.2. *The short seller's perspective.* The stock short seller has a short position in the asset and would use a long call to insure this position. Let us look at the following situation:

- the initial stock price is $S(0) = 100$; the short seller receives the proceeds of the short sale at time-0;
- the continuously compounded risk-free interest rate equals 0.04;
- the short sale is closed at time $T = 1$;
- a 100$-$strike, one-year European call on the stock is sold for \$5 at time-0.

What is the short seller's payoff curve?



What about the profit curve?