UNIVERSITY OF TEXAS AT AUSTIN

Lecture 6

Collars. Risk management using collars.

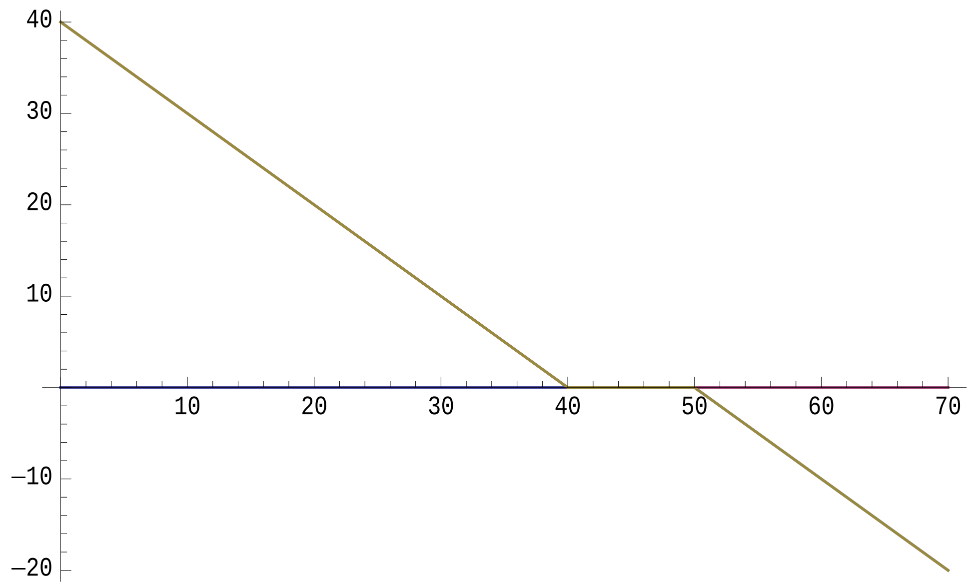
6.1. **Definition.** A *collar* is a financial position consisting of:

- the **purchase** of a put option
  and
- the **sale** of a call option with a **higher** strike price,

with both options having the same underlying asset and having the same expiration date. In short, with $K_P \leq K_C$,

$$\text{collar} = \text{long put(strike } K_P) + \text{short call(strike } K_C).$$

Here is the payoff curve of a long collar.



Example 6.1. Sample FM (Derivatives Markets): Problem #3.
Happy Jalapeños, LLC has an exclusive contract to supply jalapeño peppers to the organizers of the annual jalapeño eating contest. The contract states that the contest organizers will take delivery of 10,000 jalapeños in one year at the market price. It will cost Happy Jalapeños 1,000 to provide 10,000 jalapeños and today's market price is 0.12 for one jalapeño. The continuously compounded risk-free interest rate is 6%.

Happy Jalapeños has decided to hedge as follows (both options are one year, European):

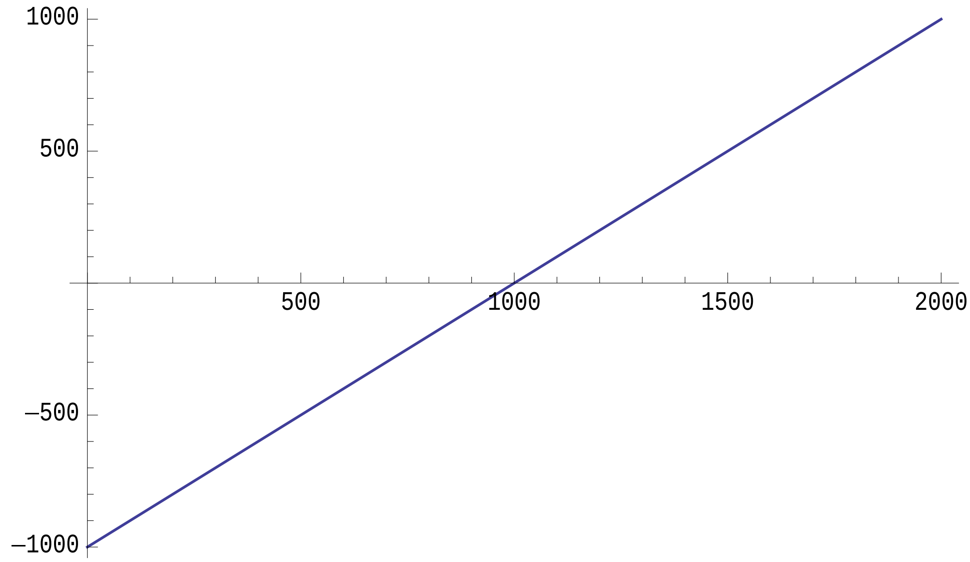
(1) buy 10,000 0.12-strike put options for 84.30, and
(2) sell 10,000 0.14-strike call options for 74.80.

Happy Jalapeños believes the market price in one year will be somewhere between 0.10 and 0.15 per pepper. Which interval represents the range of possible profit one year from now for Happy Jalapeños?

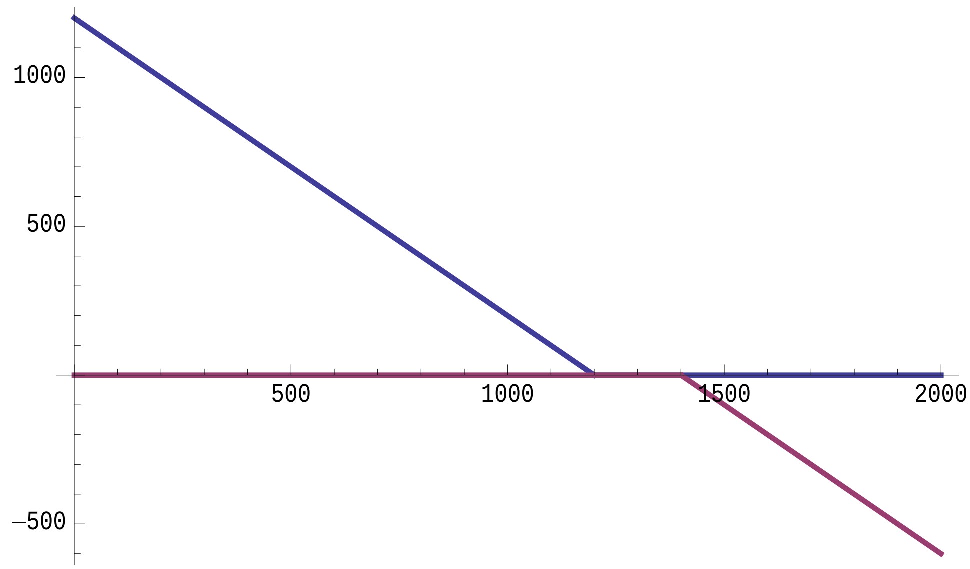
A. 200 to 100
B. 110 to 190

C. 100 to 200
D. 190 to 390
E. 200 to 400

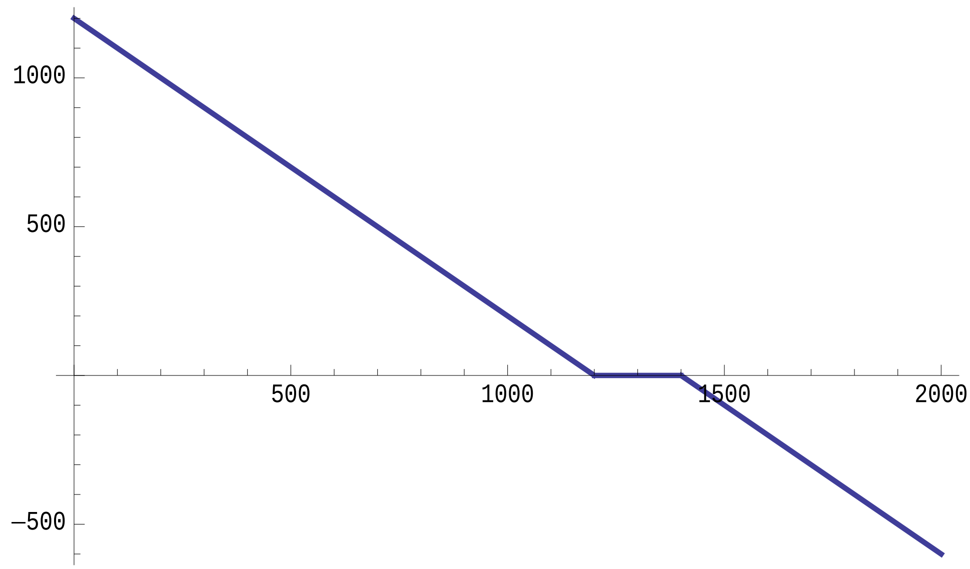
Solution: First, let's see what position the Happy Jalapeños is in before the hedging takes place. Denote the market price of 1,000 peppers in one year by $S(T)$. This means that the Happy Jalapeños will spend \$1000 for the peppers and receive $S(T)$ at delivery. So, their payoff will be $S(T) - 1000$. The graph of the payoff function is below.



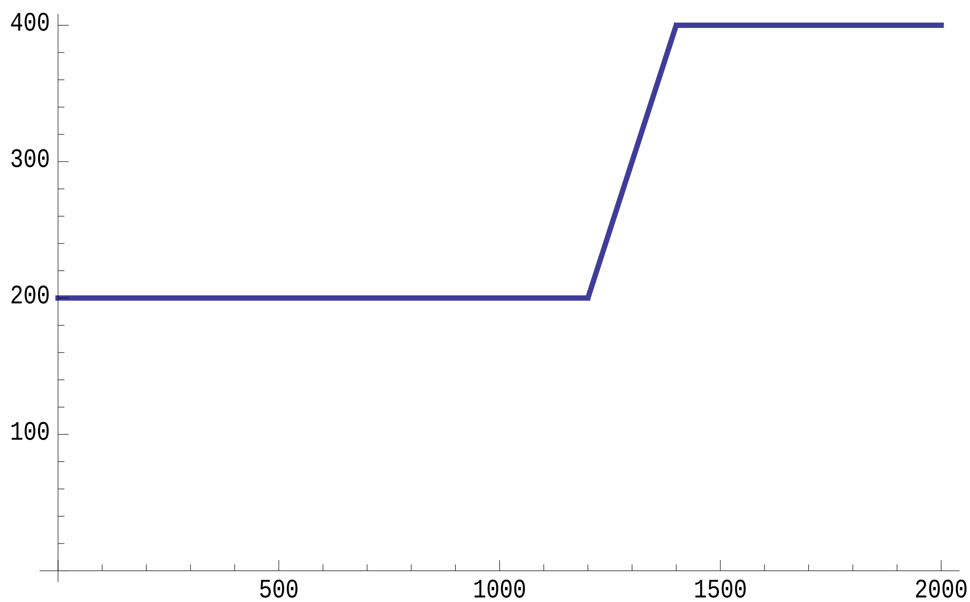
Evidently, Happy Jalapeños might be worried about low market prices of the peppers at delivery time. So, they hedge using derivatives. Let us take a look at their hedge. In the graph below, the red line indicates the payoff of the short calls, while the blue line corresponds to the payoff of the long puts.



The combined hedge position is the sum of the two payoffs depicted in the next graph.



As we can see, the particular "insurance policy" Happy Jalapeños opted for is the **collar**. Once their original position is combined with the the hedge, we get the total payoff shown in the next graph.



As we can see the payoff is bounded from below by 200 and from above by 400. This does **not** mean that we go ahead and choose the offered answer *F*. The question is about the **profit** bounds. The initial cost of the hedging position is

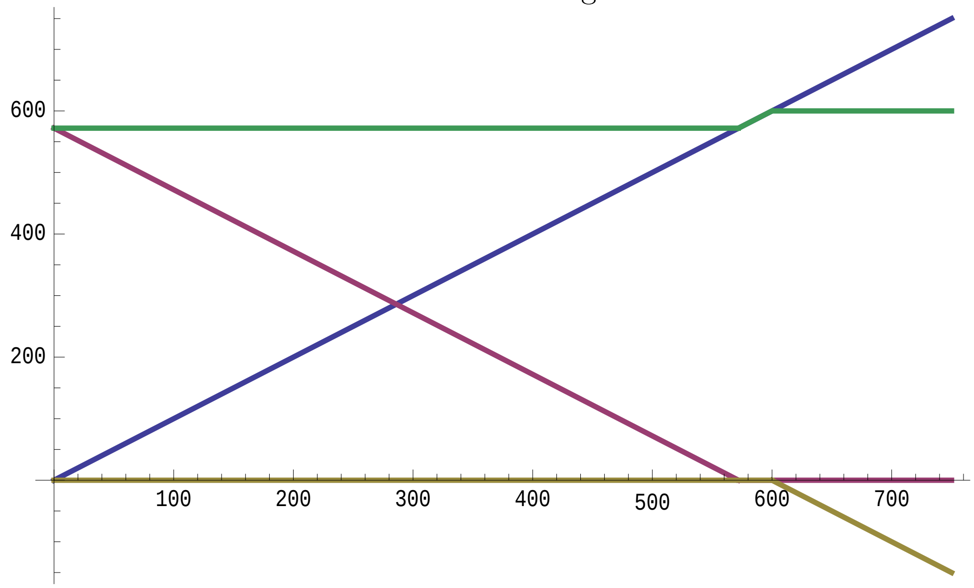
$$84.30 - 74.80 = 9.50.$$

Taking into account accrual of interest, the value at time−1 of this inital cost is

$$9.50e^{0.06} = 10.0874.$$

So, the profit lies within the interval $(200 - 10.0874, 400 - 10.0874)$. The appropriate answer is $\boxed{\text{D.}}$

FIGURE 1. Widget



6.2. **Problems.**

Problem 6.1. Widget. Min and Max profit
Source: Dr. Jim Daniel (personal communication).
The future value in one year of the total costs of manufacturing a widget is $500. You will sell a widget in one year at its market price of $S(1)$.

Assume that the annual effective interest rate equals 10%, and that the current price of the widget equals $520.

You now purchase a one-year, $572-strike put on one widget for a premium of $10. You sell some of the gain by writing a one-year, $600-strike call on one widget for a $3 premium.

What is the **range** of the profit of your hedged porfolio?

Solution: The payoff diagram for the above hedging situation is shown in Figure 1. The blue line corresponds to the unhedged position, the red line is the long-put payoff, the gold line is the short-call payoff, and the green line is the hedged portfolio payoff. As you can see, the range of the payoff is $[572, 600]$ (exactly the range between the two strikes!).

The future value of the total cost of both production and hedging is

$$500 + (10 - 3)(1 + 0.10) = 492.30.$$

So, the range of the profit equals $[64.30, 92.30]$.

Problem 6.2. Widget and verge.
Source: Dr. Jim Daniel (personal communication).
You plan to sell a widget in one year and your gain will be $\$500 - S(1)$, where $S(1)$ denote the price of an item called the *verge* (needed to complete the widget).

Assume that the effective annual risk-free interest rate equals 10%.

Your hedge consists of the following two components:

(1) one **long** one-year, \$450-strike call option on the verge whose premium is \$3.00,
(2) one **written** one-year, \$420-strike put option on the verge whose premium is \$10.00.

Calculate the profit of the hedged portoflio for the following two scenarios:

(1) the time−1 price of the verge is \$440,
(2) the time−1 price of the verge is \$475.

Solution: The hedged portfolio consists of the following components:

(1) **revenue** from the verge sales,
(2) one **long** one-year, \$450-strike call option on the verge whose premium was \$3.00,
(3) one **written** one-year, \$420-strike put option on the verge whose premium was \$10.00.

The initial cost for this portfolio is the cost of hedging (all other accumulated production costs are incorporated in the revenue expression $500 - S(1)$). Their future value is

$$(3 - 10) \times 1.10 = -7.7.$$

As usual, the negative initial cost signifies an initial influx of money for the investor.

In general, the profit expression is:

$$500 - S(1) + (S(1) - 450)_+ - (420 - S(1))_+ + 7.7.$$

So, we get the following profits in the two scenarios:

(1) the time−1 price of the verge is \$440:

$$500 - 440 + (440 - 450)_+ - (420 - 440)_+ + 7.7 = 67.70.$$

(2) the time−1 price of the verge is \$475.

$$500 - 440 + (440 - 450)_+ - (420 - 440)_+ + 7.7 = 57.70.$$

Remark 6.2. We see above that the **user/buyer** of goods uses a <u>short</u> collar to hedge.