UNIVERSITY OF TEXAS AT AUSTIN

Lecture 12

Gap options.

12.1. **Gap calls.** A European *gap call option* is a derivative security on an underlying asset (with price denoted by $\mathbf{S} = \{S(t), t \geq 0\}$) which given:

- an exercise date T;
- a **strike** price K_s;
- a **trigger** price K_t

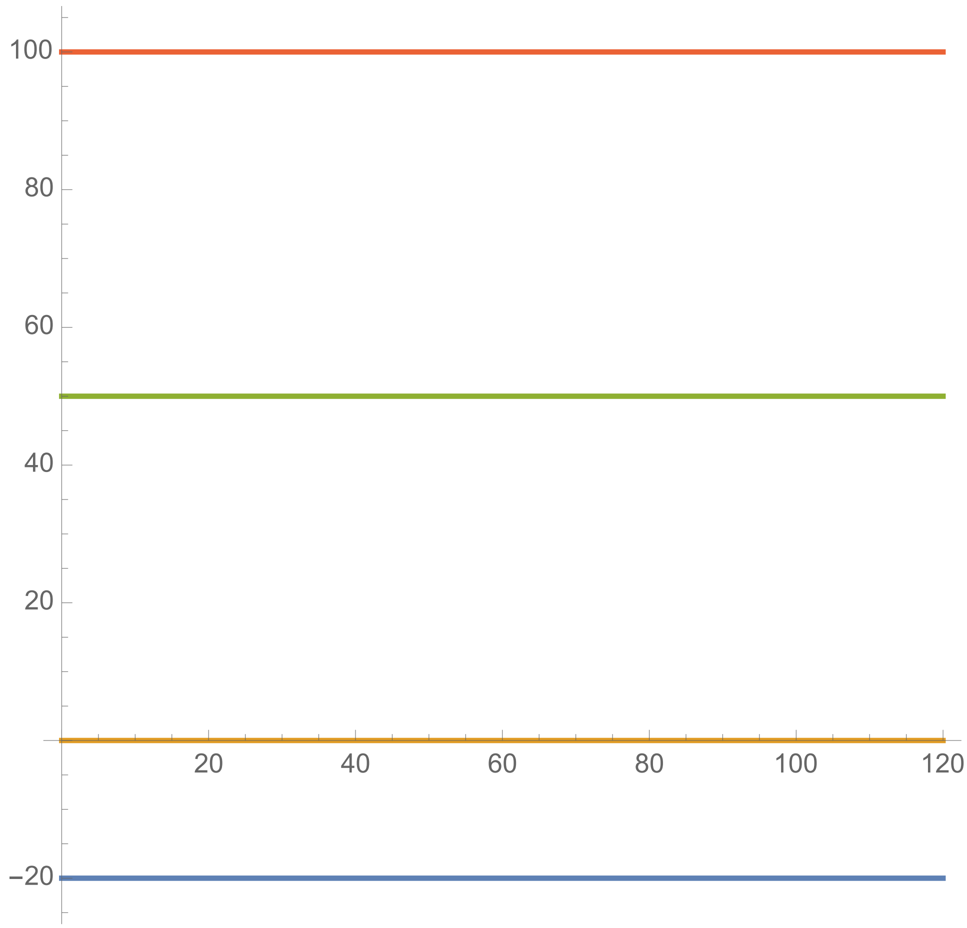
provides the payoff

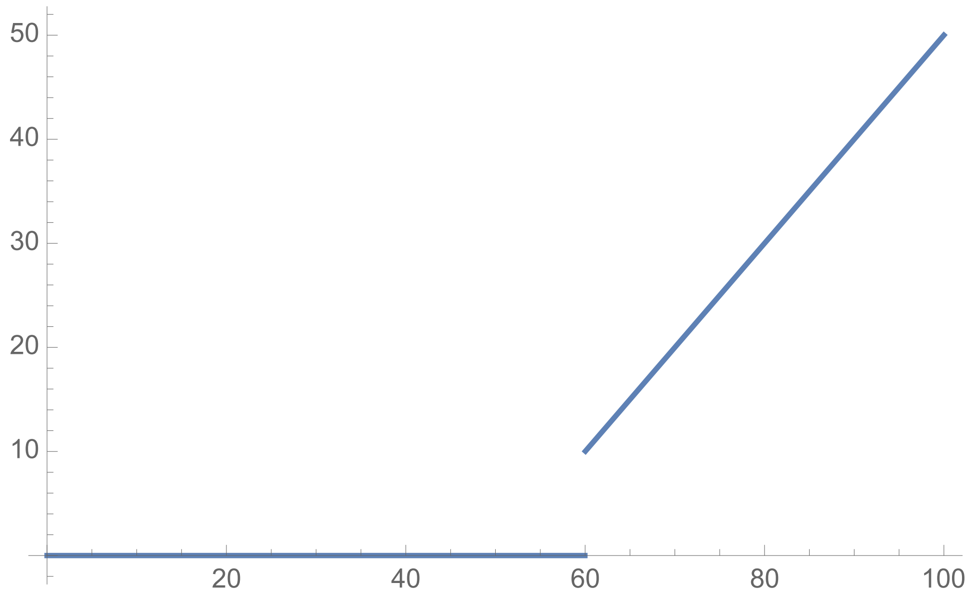
$$V_{GC}(T) = (S(T) - K_s)\mathbb{I}_{[S(T) \geq K_t]}$$

to its owner.

Problem 12.1. Consider a gap call option with $K_s \leq K_t$.

- Draw its payoff curve.
- Is a long gap call a long or a short position with respect to the underlying asset for the above ordering of the strike price and the trigger price?

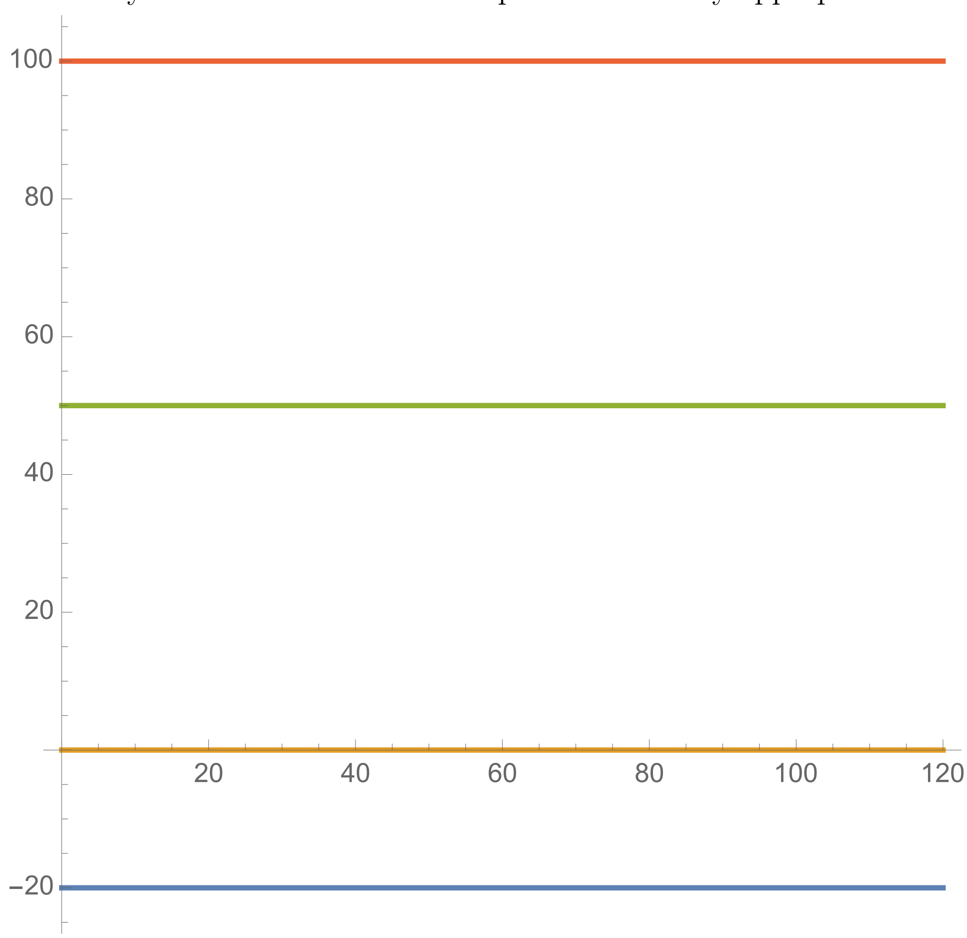


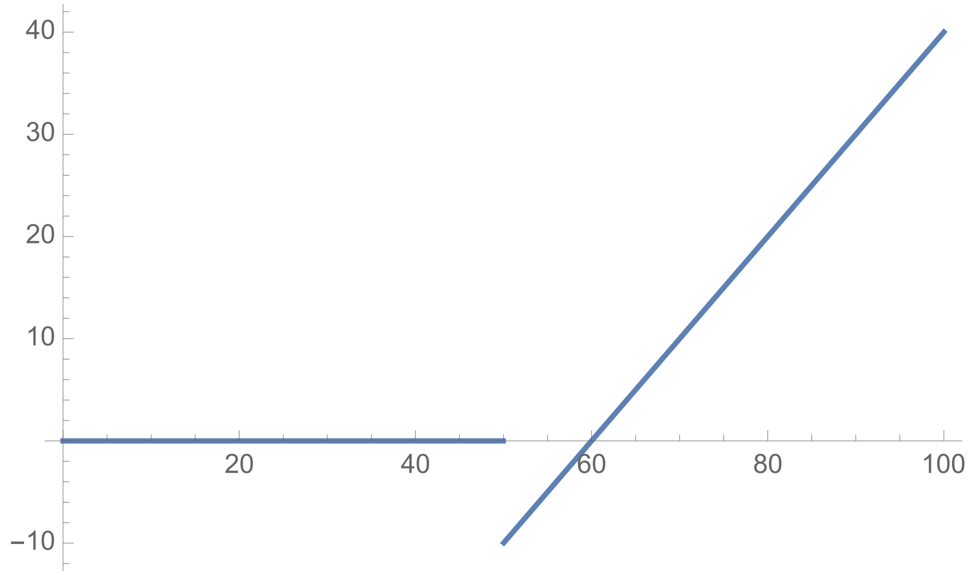


Solution:
A long position.

Problem 12.2. Consider a gap call option with $K_t < K_s$.

- Draw its payoff curve.
- Do you think that the word "option" is entirely appropriate in this case?





Solution:

A negative payoff is possible. So, the term "option" is somewhat of a misnomer.

Problem 12.3. Create a replicating portfolio for the gap call option consisting of cash-or-nothing call options and asset-or-nothing call options.

Solution:

- **long** one asset-or-nothing call with trigger price K_t,
- **short** K_s cash-or-nothing calls with trigger price K_t.

12.2. **Gap puts.** A European *gap put option* is a derivative security on an underlying asset (with price denoted by $\mathbf{S} = \{S(t), t \geq 0\}$) which given:

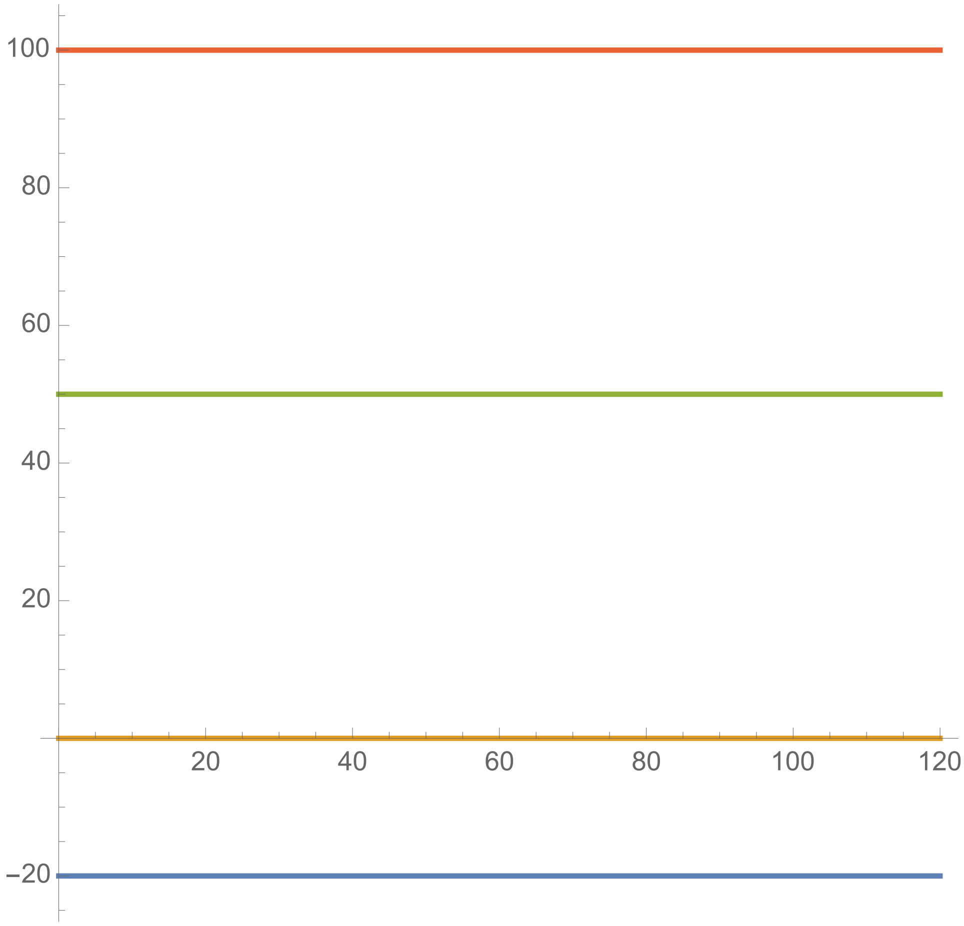
- an exercise date T;
- a **strike** price K_s;
- a **trigger** price K_t

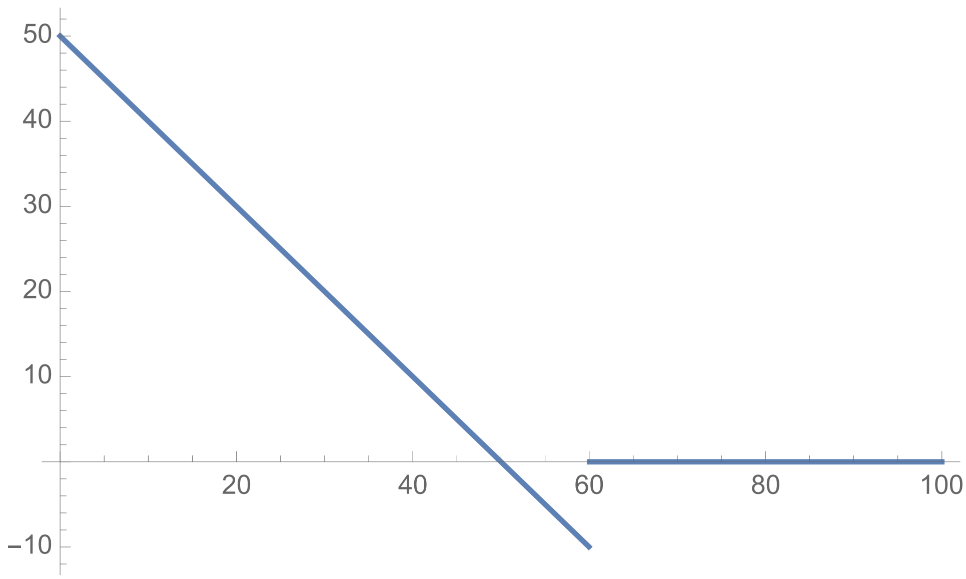
provides the payoff

$$V_{GP}(T) = (K_s - S(T))\mathbb{I}_{[S(T)<K_t]}$$

to its owner.

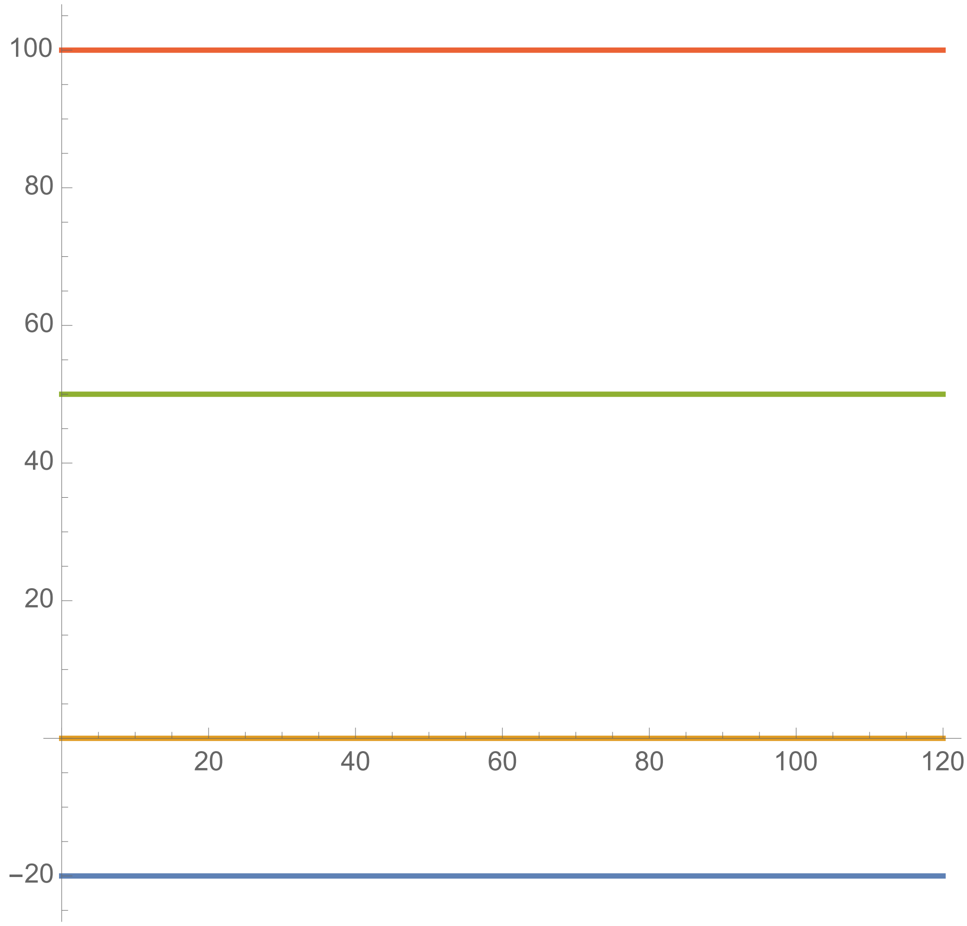
Problem 12.4. Consider a gap put option with $K_s \leq K_t$. Draw its payoff curve.

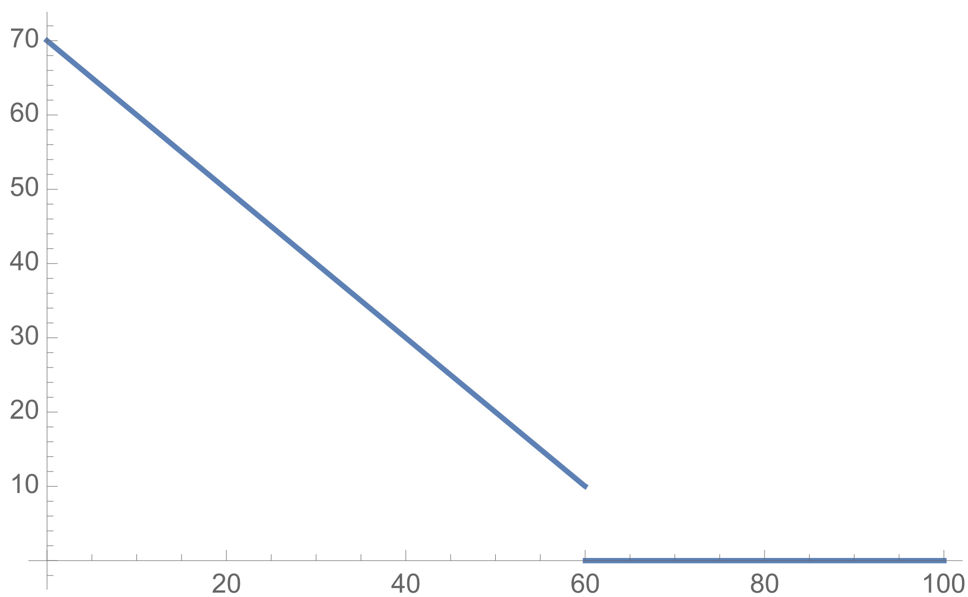




Solution:

Problem 12.5. Consider a gap put option with $K_s > K_t$. Draw its payoff curve.





Solution:

Problem 12.6. Create a replicating portfolio for the gap put option consisting of cash-or-nothing put options and asset-or-nothing put options.

Solution:

- **short** one asset-or-nothing put with trigger price K_t,

- **long** K_s cash-or-nothing puts with trigger price K_t.

12.3. **Put-call parity for gap options.**

Problem 12.7. Consider the following portfolio:

- one **long** gap call option with trigger price K_t and the strike price K_s,
- one **short** otherwise identical gap put option.

(i) What is the initial cost of the above portfolio expressed in terms of the price of the gap call $V_{GC}(0)$ and the price of the gap put $V_{GP}(0)$?
(ii) What is the payoff of the above portfolio?
(iii) Based on your answers to the above two questions, what is **put-call parity** for gap options?

Solution:

(i) $V_{GC}(0) - V_{GP}(0)$
(ii)
$$(S(T) - K_s)\mathbf{I}_{[S(T)\geq K_t]} - (K_s - S(T))\mathbf{I}_{[S(T)<K_t]} = S(T) - K_s$$
(iii)
$$V_{GC}(0) - V_{GP}(0) = F^P_{0,T}(S) - PV_{0,T}(K_s)$$

Note that the right-hand side of the parity relationship does not depend on the trigger price.