18.

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D: March 4TH, 2019.

19.

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20.

The current price of a stock is 200, and the continuously compounded risk-free interest rate is 4%. A dividend will be paid every quarter for the next 3 years, with the first dividend occurring 3 months from now. The amount of the first dividend is 1.50, but each subsequent dividend will be 1% higher than the one previously paid.

Calculate the fair price of a 3-year forward contract on this stock.

Due on Monday as a Quiz!

(A) 200

(B) 205

(C) 210

(D) 215

(E) 220

21.

$F^{obs}_{0,1/2} = 112$

A market maker in stock index forward contracts observes a 6-month forward price of 112 on the index. The index spot price is 110 and the continuously compounded dividend yield on the index is 2%. $S(0) = 110$ $\delta = 0.02$

The continuously compounded risk-free interest rate is 5%. $r = 0.05$

Describe actions the market maker could take to exploit an arbitrage opportunity and calculate the resulting profit (per index unit).

No-arbitrage f. price : $F_{0,1/2}(S) = 110\,e^{(0.05-0.02)(0.5)}$

(A) Buy observed forward, sell synthetic forward, Profit = 0.34

(B) Buy observed forward, sell synthetic forward, Profit = 0.78 ×

(C) Buy observed forward, sell synthetic forward, Profit = 1.35 ×

(D) Sell observed forward, buy synthetic forward, Profit = 0.78 ×

(E) Sell observed forward, buy synthetic forward, Profit = 0.34

$F_{0,1/2}(S) = 111.66$

1.

I. Diagnosis.

$$F^{obs}_{0,1/2} \neq F_{0,1/2}(S)$$

$\Rightarrow$ There exists an arbitrage opportunity!

II. Proposal.

$$\underbrace{F^{obs}_{0,1/2}}_{\text{relatively expensive}} > \underbrace{F_{0,1/2}(S)}_{\text{relatively cheap.}}$$

$\Downarrow$

SHORT the observed forward

$\Downarrow$

LONG the "synthetic" forward, i.e., it's sufficient to LONG $e^{-\delta \cdot T}$ shares of stock

III. Verification.

Initial cost : $S(0)e^{-\delta \cdot T}$

Payoff : $F^{obs}_{0,1/2} - S(T) + \underbrace{e^{\delta \cdot T} e^{-\delta \cdot T} \cdot S(T)}_{\text{1 share we end up owning @ time T}}$

$\Rightarrow$ Profit = Payoff $- FV_{0,T}$(Init. Cost) (2.)

$$= F^{obs}_{0,1/2} - S(0)e^{-\delta \cdot T} \cdot e^{rT} = F^{obs}_{0,1/2} - F_{0,1/2}(S) > 0$$

Q: What if $F_{0,T}(S) > F^{obs}_{0,T}$?

Then:
- LONG the OBSERVED foward
- SHORT SELL $e^{-\delta \cdot T}$ shares of the underlying

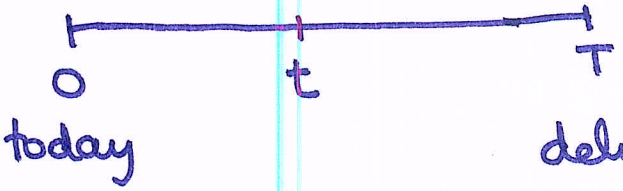
$\Rightarrow$ Profit (Arbitrage Portfolio) =

$= F_{0,T}(S) - F^{obs}_{0,T} > 0$ ∴

Futures Contracts

... standardized, liquid versions of forward contracts w/ credit risk addressed through a margin account
and w/ extra 'regulation' through the exchange

$F_{0,T}$ ... futures price @ time-0 for delivery @ time-T



$F_{t,T}$ ... the futures price @ time-t for delivery @ time-T

(t: valuation date)

Futures prices and forward prices coincide if we have a contant deterministic interest rate.

e.g., for futures on a market index, we would have

$$F_{t,T}(S) = S(t)\, e^{(r-\delta)\cdot(T-t)}$$

(S: market index)

→ Margin Account

* **Notional value:** (N)

$N = \#\text{ of contract} \times \#\text{ of units per contract} \times \text{Price per unit}$

* **Margin account** : think about it as a savings account which earns interest (in full generality, this could be a floating interest rate)

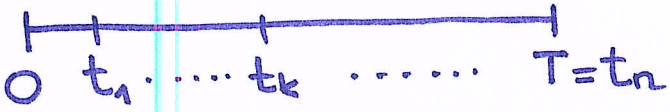
But: In our exam, there is just the single, deterministic ccrfir (r)

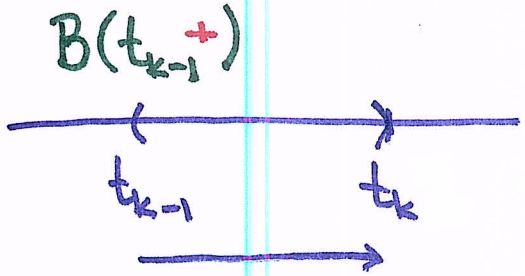
* **Initial margin**:

$$\underbrace{B^b(0)}_{\text{the buyer's balance (the long futures)}} = \underbrace{B^s(0)}_{\text{the seller's balance (the short futures)}} = q \cdot N$$

(q: a percentage; N: notional value)

* **Settlement times** (usually daily)





$$B(t_k^-) = B(t_{k-1}^+)\, e^{r(t_k - t_{k-1})}$$ ... just earning interest

* marking-to-market *

At t_k the futures price is observed

$F_{t_k,T}$

$$B^b(t_k) = B^b(t_k^-) + \#\text{of contracts} \times \text{Size} \times \left(F_{t_k,T} - F_{t_{k-1},T}\right)$$

$$B^s(t_k) = B^s(t_k^-) - \#\text{of contracts} \times \text{Size} \times \left(F_{t_k,T} - F_{t_{k-1},T}\right)$$

* The Maintenance Margin (MM)

... the minimum acceptable balance (for the broker) in the margin account.

If $B(t_k) < MM$, then a MARGIN CALL is issued. Then, sufficient funds are deposited into the margin account to go back to the initial margin.

$$\Rightarrow \quad B(t_k^+) = \begin{cases} B(t_k) & \text{if } B(t_k) \geq MM \\ \text{Initial margin} = B(0), & \text{if } B(t_k) < MM \end{cases}$$

30.

Determine which of the following is NOT a distinguishing characteristic of futures contracts, relative to forward contracts.

(A) Contracts are settled daily, and marked-to-market.

(B) Contracts are more liquid, as one can offset an obligation by taking the opposite position.

(C) Contracts are more customized to suit the buyer's needs.

(D) Contracts are structured to minimize the effects of credit risk.

(E) Contracts have price limits, beyond which trading may be temporarily halted.

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32.

Judy decides to take a short position in 20 contracts of S&P 500 futures. Each contract is for the delivery of 250 units of the index at a price of 1500 per unit, exactly one month from now. The initial margin is 5% of the notional value, and the maintenance margin is 90% of the initial margin. Judy earns a continuously compounded risk-free interest rate of 4% on her margin balance. The position is marked-to-market on a daily basis.

On the day of the first marking-to-market, the value of the index drops to 1498. On the day of the second marking-to-market, the value of the index is X and Judy is not required to add anything to the margin account.

Calculate the largest possible value of X.

(A) 1490.50

(B) 1492.50

(C) 1500.50

(D) 1505.50

(E) 1507.50

69.

Determine which of the following statements about futures and forward contracts is false.

(A) Frequent marking-to-market and settlement of a futures contract can lead to pricing differences between a futures contract and an otherwise identical forward contract.

(B) Over-the-counter forward contracts can be customized to suit the buyer or seller, whereas futures contracts are standardized.

(C) Users of forward contracts are more able to minimize credit risk than are users of futures contracts.

(D) Forward contracts can be used to synthetically switch a portfolio invested in stocks into bonds.

(E) The holder of a long futures contract must place a fraction of the cost with an intermediary and provide assurances on the remaining purchase price.

70.

Investors in a certain stock demand to be compensated for risk. The current stock price is 100.

The stock pays dividends at a rate proportional to its price. The dividend yield is 2%.

The continuously compounded risk-free interest rate is 5%.

Assume there are no transaction costs.

Let X represent the expected value of the stock price 2 years from today. Assume it is known that X is a whole number.

Determine which of the following statements is true about X.

(A) The only possible value of X is 105.

(B) The largest possible value of X is 106.

(C) The smallest possible value of X is 107.

(D) The largest possible value of X is 110.

(E) The smallest possible value of X is 111.