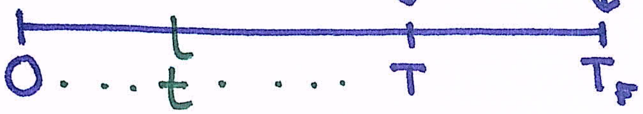
D: March 15th, 2019.

Options on Futures.

$\underbrace{\text{Calls / Puts}}_{\substack{\text{expiration} \\ \text{date } T}}$ on $\underbrace{\text{Futures Contracts}}_{\substack{\text{delivery} \\ \text{date } T_F}}$



Always: $T \leq T_F$

It's possible to have $T = T_F$ (in particular, if the option is <u>American</u>).

If the option is exercised @ time-t:

- IF it's a call, <u>then</u> the owner of the call:
  - * takes a <u>long</u> position in the underlying futures contract
  - * gets $(F_{t,T_F} - K)_+$

- IF it's a put, <u>then</u> the owner of the put:
  - * takes a <u>short</u> position in the underlying futures contract
  - * gets $(K - F_{t,T_F})_+$

Focus on a pair of a European call & put

- on the same futures contract
- w/ strike K, and
- w/ exercise date T

Build the following portfolio:

- <u>LONG</u> futures call
- <u>WRITTEN</u> futures put

⇒ The portfolio's payoff is

$$(F_{T,T_F} - K)_+ - (K - F_{T,T_F})_+ =$$

$$= \begin{cases} F_{T,T_F} - K & \text{if } F_{T,T_F} \geq K \\ -K + F_{T,T_F} & \text{if } F_{T,T_F} < K \end{cases}$$

$$= F_{T,T_F} - K$$

w/ a <u>LONG</u> position in the futures contract.

Temporarily, focus on futures on market indices.

<u>In this case</u>: Futures prices = Forward prices

$$\Rightarrow \boxed{F_{T,T_F}(S) = \underbrace{S(T)}\, e^{(r-\delta)(T_F-T)}}$$

constant: can be interpreted as a number of shares

$\Rightarrow$ The time-0 cost of the above # of shares to be delivered @ time T is

$$F^P_{0,T}(S)\cdot e^{(r-\delta)(T_F-T)} =$$

$$= S(0)e^{-\delta\cdot T}e^{(r-\delta)(T_F-T)}$$

$$= \underbrace{S(0)\cancel{e^{-\delta\cdot T}}\cdot e^{(r-\delta)\cdot T_F}}\cdot e^{-r\cdot T}\cdot \cancel{e^{\delta\cdot T}}$$

$$= F_{0,T_F}(S) \cdot e^{-r\cdot T}$$

$\Rightarrow$ In this case:

$$V_C(0) - V_P(0) = F_{0,T_F}(S)\cdot e^{-r\cdot T} - Ke^{-r\cdot T}$$

We will generalize the above to all futures contracts. We get the put-call parity

$$V_C(0) - V_P(0) = e^{-r\cdot T}(F_{0,T_F} - K)$$

Recall the "vanilla" put·call parity for options on cont·div·paying stocks:

$$V_C(0) - V_P(0) = F^P_{0,T}(S) - PV_{0,T}(K)$$

$$= S(0)e^{-\delta \cdot T} - Ke^{-r \cdot T}$$

Analogy: futures contracts $\leftrightarrow$ cont. dividend stocks

$$r \leftrightarrow \delta$$

Note: Call & Put are @·the·money

$$\Longleftrightarrow$$

$$V_C(0) = V_P(0)$$

Option on Currencies

→ underlying asset... foreign currency (FC)

→ asset price ... the EXCHANGE RATE:

$x(t),\ t \geq 0$ (from FC to DC)

- r_D ... ccrfir for DC
- r_F ... ccrfir for FC

Prepaid forward

Prepaid Forward: $F^P_{0,T}(x)$ 1 unit of FC

$\parallel$

Outright Purchase: buy $e^{-r_F \cdot T}$ units 1 unit of FC

cost: $x(0)e^{-r_F \cdot T}$

$\Rightarrow \quad F^P_{0,T}(x) = x(0)e^{-r_F \cdot T}$ ... DC-denominated

$\Rightarrow \quad F_{0,T}(x) = x(0)e^{(r_D - r_F)\cdot T}$

Recall: continuous dividend paying stocks

$$F^P_{0,T}(S) = S(0)\,e^{-\delta \cdot T}$$

$$F_{0,T}(S) = S(0)\,e^{(r-\delta)\cdot T}$$

$\Rightarrow$ **Analogy:** exchange rate $\longleftrightarrow$ cont. dividend stocks

$r_F \longleftrightarrow \delta$

$r_D \longleftrightarrow r$

For European calls & put

w/ the same strike K & exercise date T:

Payoffs: $V_c(T) = (x(T) - K)_+$

and $V_p(T) = (K - x(T))_+$

Goal: Put-call Parity for currency options.

Build: $\left\{\begin{array}{l} \cdot \text{ long call} \\ \cdot \text{ written put} \end{array}\right.$

$\Rightarrow$ Payoff of the portfolio is $x(T) - K$.

$$\Rightarrow \boxed{V_c(0) - V_p(0) = x(0)e^{-r_F \cdot T} - Ke^{-r_D \cdot T}}$$

Put-Call Parity