D: March 25th, 2019.

vanilla calls/puts

shares $\downarrow$ $\uparrow$ cash

$\Rightarrow$ Call Payoff:

$(S(T)-K)_+$

Put Payoff:

$(K-S(T))_+$

currency options

FC $\downarrow$ $\uparrow$ DC

$\Rightarrow$ Call Payoff:

$(x(T)-K)_+$

Put Payoff:

$(K-x(T))_+$

Introduce an option type:

stock #1 $\downarrow$ $\uparrow$ stock #2

EXCHANGE OPTIONS

... calls/puts where BOTH the underlying and the strike are RISKY ASSETS.

Notation: S and Q denote the two assets

For now: $\begin{cases} S \dots \text{underlying asset} \\ Q \dots \text{strike asset} \end{cases}$

Exchange call: A right, but not an obligation,
to receive 1 share of S
and give up 1 share of Q.

$\Rightarrow$ The payoff of the exchange call is

$$V_{EC}(T, \underbrace{S}_{\text{underlying}}, \underbrace{Q}_{\text{strike asset}}) := (S(T) - Q(T))_+$$

Exchange put: A right, but not an obligation, to
give up 1 share of S
and receive 1 share of Q.

$\Rightarrow$ The payoff is:

$$V_{EP}(T, S, Q) := (Q(T) - S(T))_+$$

$\Rightarrow$ A special symmetry: $V_{EC}(T, S, Q) = V_{EP}(T, Q, S)$

$\Rightarrow$ for any $0 \le t \le T$:

$$V_{EC}(t, S, Q) = V_{EP}(t, Q, S)$$

Maximum Options

$$\left.\begin{array}{l}\{S(t), t \geq 0\} \\ \{Q(t), t \geq 0\}\end{array}\right\} \text{ RISKY ASSETS}$$

Set the payoff of the maximum option:

$$Y_{MAX}(T) := MAX(S(T), Q(T))$$

Q: Can you come up w/ a financial story for the implementation of this payoff?

→: The owner of the option gets to receive either one share of S, or one share of Q. ∎

Q: Bounds on the price of the maximum option?

$$\rightarrow: \quad Y_{MAX}(T) \geq \begin{cases} S(T) \\ Q(T) \end{cases}$$

$$\Rightarrow Y_{MAX}(0) \geq \begin{cases} F^P_{0,T}(S) \\ F^P_{0,T}(Q) \end{cases}$$

$$\Rightarrow V_{MAX}(0) \geq MAX\left(F^P_{0,T}(S), F^P_{0,T}(Q)\right)$$

Q: Replicating portfolio?

$\rightarrow$: $V_{MAX}(T) = MAX(S(T), Q(T))$

$$= \underline{\underline{S(T)}} + MAX(0, Q(T) - S(T)) \quad \text{PAYOFF of an EXCHANGE OPTION!}$$

$$= \underline{\underline{Q(T)}} + MAX(S(T) - Q(T), 0)$$

$\Rightarrow$ Our maximum option can be replicated w/:

- one LONG prepaid forward on S
- one LONG exchange $\underline{\underline{\text{call}}}$ w/ underlying $\underline{Q}$ and strike asset $\underline{S}$

$\Rightarrow$ $V_{MAX}(0) = F^P_{0,T}(S) + V_{EC}(0, Q, S)$

Similarly:

$$\boxed{\begin{aligned} &= F^P_{0,T}(Q) + \underline{V_{EC}(0, S, Q)} \\ &= F^P_{0,T}(S) + \underline{V_{EP}(0, S, Q)} \end{aligned}}$$

$$= F^P_{0,T}(Q) + V_{EP}(0, Q, S)$$

$$\boxed{V_{EC}(0, S, Q) - V_{EP}(0, S, Q) = F^P_{0,T}(S) - F^P_{0,T}(Q)}$$

Generalized put-call parity

6. Consider a model with two stocks. Each stock pays dividends continuously at a rate proportional to its price.

$S_j(t)$ denotes the price of one share of stock j at time t.

Consider a claim maturing at time 3. The payoff of the claim is

$$\text{Maximum}\,(S_1(3), S_2(3))$$ ... our maximum option

You are given:

(i) $S_1(0) = \$100$

(ii) $S_2(0) = \$200$

(iii) Stock 1 pays dividends of amount $(0.05)S_1(t)dt$ between time t and time $t+dt$. $\delta_1 = 0.05$

(iv) Stock 2 pays dividends of amount $(0.1)S_2(t)dt$ between time t and time $t+dt$. $\delta_2 = 0.10$

(v) The price of a European option to exchange Stock 2 for Stock 1 at time 3 is \$10.

exercise date

Calculate the price of the claim.

(A) \$96

(B) \$145

(C) \$158

(D) \$200

(E) \$234

An exchange call w/ underlying S_1 and strike asset S_2

$\Rightarrow V_{EC}(0, S_1, S_2) = 10$

$$V_{MAX}(0) = F^P_{0,3}(S_2) + V_{EC}(0, S_1, S_2)$$
$$= 200e^{-0.1(3)} + 10$$
$$\cong 158 \Rightarrow (C)$$

5.

Q: $Y_{min}(T) = \min(S(T), Q(T))$

$$\dots = S(T) + \min(0, Q(T) - S(T))$$

$$= S(T) - \underbrace{\max(0, S(T) - Q(T))}_{\text{exchange option}}$$

shorting the option

Q: Payoff of a special option w/

$$\max(\alpha \cdot S(T), \beta \cdot Q(T))$$

α: const. β: const.

:)

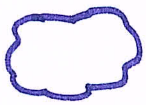
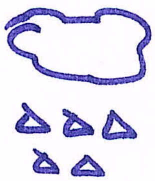
Q: What is the payoff of a special option which allows its owner to buy 2 shares of S_1 or 3 shares of S_2 for the strike price K?

$$\left(\max(2S_1(T), 3S_2(T)) - K\right)_+$$

Q: Change "buy" to "sell" above!

$$\left(K - \min(2S_1(T), 3S_2(T))\right)_+$$

Q :

	$S(T)$	$Q(T)$	
	150	90	w/ p_1
	100	105	w/ p_2
	80	200	w/ p_3

$\mathbb{E}$[Payoff of any of the options we saw today]