# Commodity Swaps

May 7th

**Recall:** $\boxed{\text{Forwards}}$

$0$ ——————— $T$

Delivery date

**In general w/ SWAPS:**

$x_1 \quad x_2 \quad \cdots \quad x_k \quad \cdots \quad x_n$ ← SWAP PRICES

$0 \quad T_1 \quad T_2 \quad \cdots \quad T_k \quad \cdots \quad T_n$ ← SWAP DATES

**On any swap date $T_k$:**

If Physical Settlement:

BUYER

$x_k \downarrow \quad \uparrow$ 1 unit of the asset

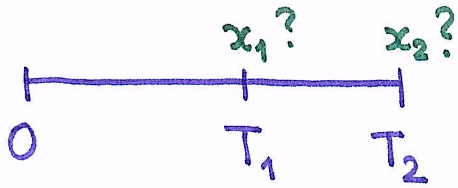
SELLER

The NET-EFFECT for the buyer:

$$\boxed{\underbrace{S(T_k)}_{\text{the market price of the asset}} - x_k}$$

CASH SETTLEMENT is feasible

Focus on <u>two</u> swap dates (generalization is easy :) ).




Q: What are the $\boxed{\text{ACCEPTABLE}}$ pairs of $\underset{\sim}{x_1}$ and $\underset{\sim}{x_2}$?

The ones which do NOT introduce arbitrage opportunities!

<u>Existing environment</u>:

- FORWARD PRICES: $F_{0,T_1}$ and $F_{0,T_2}$
- TERM STRUCTURE of interest rate, i.e.,

$$\left.\begin{array}{l} r_0(0, T_1) \\ r_0(0, T_2) \end{array}\right\} \text{YEARLY } \underline{\text{EFFECTIVE}} \text{ SPOT RATES}$$

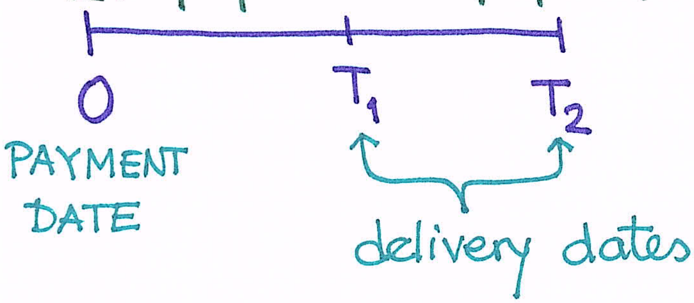
reference time

the start of the investment period

$\boxed{\text{the end of the investment period}}$

## Prepaid swaps (auxilliary contracts)

$x^P$... prepaid-swap price




A replicating portfolio for a prepaid swap is:

- a prepaid forward w/ delivery date $T_1$
- — || — — || — — || — — || — $T_2$

$$\Rightarrow \quad x^P = F^P_{0,T_1} + F^P_{0,T_2} = PV_{0,T_1}(F_{0,T_1}) + PV_{0,T_2}(F_{0,T_2})$$

NO ARBITRAGE!

$$\Rightarrow \quad x^P = \frac{F_{0,T_1}}{(1+r_0(0,T_1))^{T_1}} + \frac{F_{0,T_2}}{(1+r_0(0,T_2))^{T_2}}$$

Example. Oil delivery @ $T_1 = 1$ and $T_2 = 2$.
The forward prices for oil delivery in one year and in two years are \$20 and \$21, resp.
The effective annual yield rates for zero-coupon bonds redeemable in one year and in years are 6% and 6.5%, resp.
What is the prepaid-swap price for oil delivery in one year and in two years?

$$x^P = \frac{F_{0,1}}{(1+r_0(0,1))^1} + \frac{F_{0,2}}{(1+r_0(0,2))^2} = \frac{20}{1.06} + \frac{21}{(1.065)^2} = 37.383$$

Credit risk.
Cash availability $\Big\}$ $\Rightarrow$ "Regular" swaps are more "appealing" than their prepaid counterparts.

In general: $x_1$ and $x_2$ must satisfy

$$x^P = \frac{x_1}{(1+r_0(0,T_1))^{T_1}} + \frac{x_2}{(1+r_0(0,T_2))^{T_2}}$$

Level Swaps $\quad x$... the level swap price

$$\Rightarrow \quad x^P = x \cdot \left[\frac{1}{(1+r_0(0,T_1))^{T_1}} + \frac{1}{(1+r_0(0,T_2))^{T_2}}\right]$$

Example [cont'd]

For the oil swap : $x^P = 37.383$.

$\Rightarrow$ The level swap price is:

$$x = \frac{37.383}{\frac{1}{1.06} + \frac{1}{(1.065)^2}} = \dots = 20.483$$

NOTE: $\quad F_{0,1} = 20 < x = 20.483 < F_{0,2} = 21$

Generally: $\quad F_{0,T_1} \leq x \leq F_{0,T_n}$

The effect: Overpayment at time-$T_1$ and underpayment at time-$T_2$

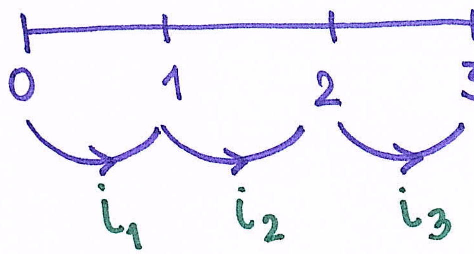
The interpretation: A LOAN from the buyer to the seller @ $r_0(1,2)$ IMPLIED FORWARD RATE FOR [1,2]

# Why Swap Interest Rates?

- Interest rate swaps permit firms to separate credit risk and interest rate risk
  - By swapping its interest rate exposure, a firm can pay the short-term interest rate it desires, while the long-term bondholders will continue to bear the credit risk

# Interest-rate swap

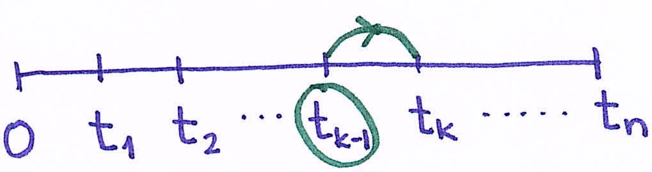
**Recall:** Time-varying interest rates




effective rates

Random, time-varying interest rates, aka FLOATING INTEREST RATES $\Rightarrow$ swaps on interest rates.

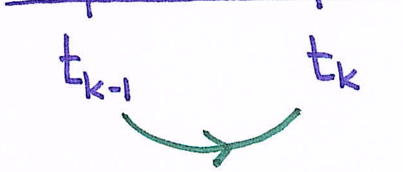
$L$ .... loan amount




Charged a floating interest rate:

$$R_{t_{k-1}}(t_{k-1}, t_k)$$

What is the payment amount to be paid @ $t_k$?




The adjustment of principal

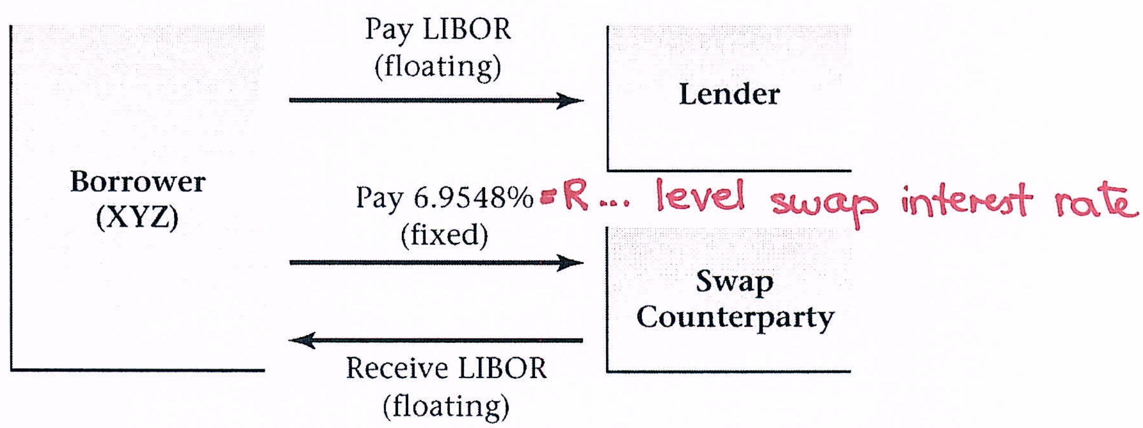
Interest Portion

The pmt amt: $A_{t_k} = P_{t_k} + I_{t_k}$

THE NOTIONAL PRINCIPAL

w/ $I_{t_k} = OLB_{t_{k-1}} \cdot R_{t_{k-1}}(t_{k-1}, t_k)$

# An Example of an Interest Rate Swap (cont'd)




- On net, XYZ pays 6.9548%

$$\text{XYZ net payment} = \underbrace{-\,\text{LIBOR}}_{\text{Floating Payment}} + \underbrace{\text{LIBOR} - 6.9548\%}_{\text{Swap Payment}} = -6.9548\%$$

## The simplest (non-trivial) case

INTEREST-ONLY PMTS

$\Rightarrow$ The notional principal:

$$\boxed{OLB_{t_k} = L}$$

NPV (all pmts using R) = NPV (all pmts using implied forward rates)

$$\sum_{k=1}^{n}\Big[(\cancel{L}\cdot R)\underbrace{P(0,t_k)}_{\text{zero-coupon bond prices}}\Big] = \sum_{k=1}^{n}\Big[(\cancel{L}\, r_0(t_{k-1},t_k))\cdot P(0,t_k)\Big]$$

$$\Rightarrow R = \frac{\sum_{k=1}^{n} r_0(t_{k-1},t_k)\, P(0,t_k)}{\sum_{k=1}^{n} P(0,t_k)}$$

$$\Rightarrow R = \sum_{k=1}^{n}\left(\frac{P(0,t_k)}{\sum_{k=1}^{n} P(0,t_k)}\cdot r_0(t_{k-1},t_k)\right)$$

For yearly installments:

$$R = \frac{1-P(0,t_n)}{\sum_{k=1}^{n} P(0,t_k)}$$

Solve Sample FM (DM) Problem #23.