Nov 20th

Asian Options [cont'd]

$$(S(t_1), \dots, S(t_k), \dots, S(t_n)) \longmapsto \begin{cases} A(T) := \frac{1}{n} \sum_k S(t_k) \\ G(T) = \left(\prod_k S(t_k) \right)^{1/n} \end{cases}$$

$$\boxed{A(T) \geq G(T)}$$

PAYOFFS		GEOMETRIC		ARITHMETIC
STRIKE	Call:	$(S(T)-G(T))_+$	$\geq$	$(S(T)-A(T))_+$
	Put:	$(G(T)-S(T))_+$	$\leq$	$(A(T)-S(T))_+$
UNDERLYING ASSET [given a strike K]	Call:	$(G(T)-K)_+$	$\leq$	$(A(T)-K)_+$
	Put:	$(K-G(T))_+$	$\geq$	$(K-A(T))_+$

$\Downarrow$

Draw analogous inequalities w/ option prices.

Problem. Non-dividend-paying stock has $S(0) = 100$.

$\sigma = 0.3$

$r = 0.04$

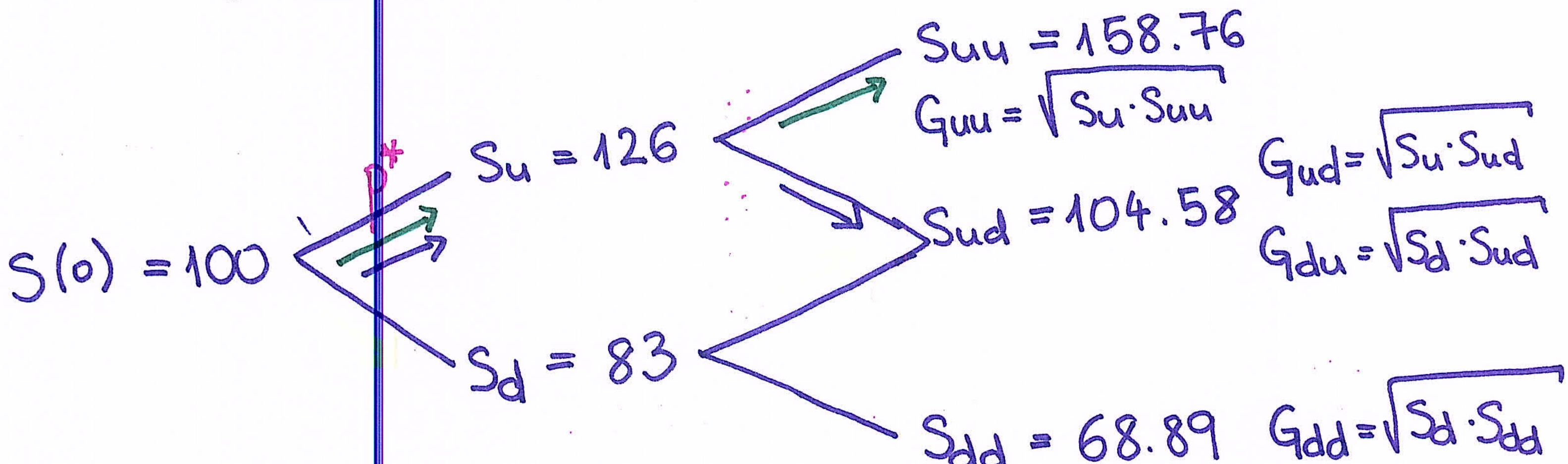
We model the stock price over the next year with a two-period FORWARD binomial tree.

$u = e^{(r-\delta)h + \sigma\sqrt{h}}$

$d = e^{(r-\delta)h - \sigma\sqrt{h}}$ w/ $h = 1/2$ and $\delta = 0$.

$\Rightarrow \quad u = e^{0.04 \cdot 0.5 + 0.3\sqrt{0.5}} \approx 1.26$

$d = e^{0.02 - 0.3\sqrt{0.5}} \approx 0.83$



$$p^* = \frac{1}{1+e^{\sigma\sqrt{h}}} = \frac{1}{1+e^{0.3\sqrt{0.5}}} \approx 0.447$$

Consider an Asian geometric-average-strike call w/ exercise date in one year.
What is its price consistent w/ the above stock-price tree?

PAYOFF: $(S(T) - G(T))_+$

<u>up-up</u> : $V_{uu} = (S_{uu} - G_{uu})_+ = \left(S(0)\cdot u^2 - \sqrt{S(0)\cdot u \cdot S(0)\cdot u^2}\right)_+$

$= S(0)\cdot u\,(u - \sqrt{u})_+ = \dots = 17.33$

<u>up-down</u> : $V_{ud} = (S_{ud} - G_{ud})_+ = S(0)\cdot u\,(d - \sqrt{d})_+ = 0$

<u>down-up</u> : $V_{du} = (S_{ud} - G_{du})_+ = \left(S(0)\cdot u\cdot d - \sqrt{S(0)\cdot d\cdot S(0)\cdot u\cdot d}\right)_+$

$= S(0)\cdot d\cdot(u - \sqrt{u})_+ = \dots = 11.41$

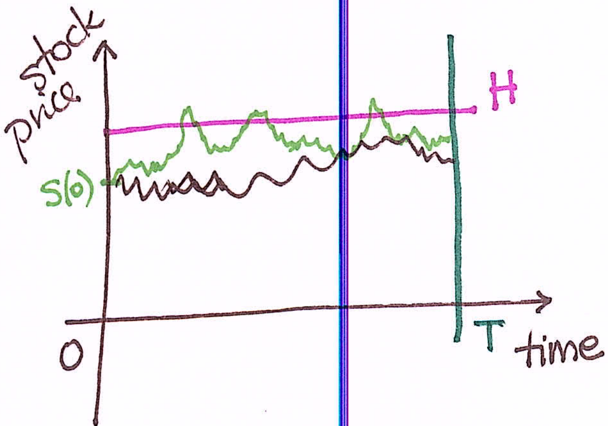
<u>down-down</u> : $V_{dd} \equiv 0$

$$V(0) = e^{-0.04}\left[(0.447)^2\cdot 17.33 + 0.447(1-0.447)\cdot 11.41\right]$$

$$\boxed{V(0) = 6.04}$$

Barrier Options

level / threshold / hurdle (H)



If the stock price touches/crosses the barrier H before or @ the exercise date, then the barrier option changes its STATE.

"Dead" → "Alive"
RIP → ♡

KNOCK IN

⇓

THE PAYOFF is paid on the exercise date T.

"Alive" → "Dead"
♡ → RIP

KNOCK OUT

⇓

PAYOFF = 0

Example. $\boxed{\text{REBATE OPTIONS}}$

A fixed amount R is paid to the option's owner at time-T if the barrier H is ever touched / crossed by the stock price.

- $M(T) = \max_{0 \le t \le T} S(t)$
- $m(T) = \min_{0 \le t \le T} S(t)$