Def'n. Consider a European-style derivative security. A static portfolio is said to be a **REPLICATING PORTFOLIO** for this derivative security if their PAYOFFS are EQUAL.

Q: Create a replicating portfolio for a forward contract using a call and a put on the same underlying!

The payoff of the forward: $S(T) - F_{0,T}(S)$

- LONG call
- \+
- SHORT put

w/ strike $\boxed{K = F_{0,T}(S)}$ and exercise date T

- What is the initial cost of this replicating portfolio? Replicates a forward
  $\Rightarrow$ has the same initial cost as a forward
  $\Rightarrow$ cost $= 0$.

Alternatively:

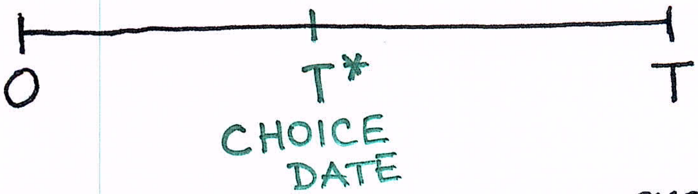
PUT-CALL PARITY

$$V_C(0) - V_P(0) = F^P_{0,T}(S) - PV_{0,T}(K)$$
$$= F^P_{0,T}(S) - PV_{0,T}(F_{0,T}(S)) = 0 \checkmark$$

Q: Does the replicating portfolio need to be unique?
Absolutely not!

- Long prepaid forward contract w/ delivery @ T
- Borrow $F^P_{0,T}(S)$ @ the risk-free rate to be repaid @ time-T.

CHOOSER OPTIONS (As-you-like-it options)



written up
- T^*
- T
- K

exercise date
the payoff happens here

The owner of the CH option decides if the option is to continue its life as a CALL or a PUT w/ strike K.

$V_{CH}(0, T^*, T, K) = ?$

valuation date

<u>First</u>: What is the optimal rational way to choose @ time-T^*?

Imagine you decide to sell the chooser option @ time-T^*. What's the price you can get for it?

$$\max\left(V_c(\underbrace{T^*}_{\text{valuation date}}, \underbrace{T}_{\text{exercise date}}, \underbrace{K}_{\text{strike}}), V_p(T^*, T, K)\right) =$$

$$= V_{CH}(T^*, T^*, T, K)$$

$$\max(a,b) = a + \max(0, b-a) = a + (b-a)_+ \quad \checkmark$$
$$= b + (a-b)_+$$

$\Rightarrow$

$$V_{CH}(T^*, T^*, T, K) = \max\left(V_c(T^*, T, K), V_p(T^*, T, K)\right)$$
$$= V_c(T^*, T, K) + \max\left(0, V_p(T^*, T, K) - V_c(T^*, T, K)\right)$$
$$= V_c(T^*, T, K) + \underbrace{\left(V_p(T^*, T, K) - V_c(T^*, T, K)\right)}_{\text{PUT-CALL PARITY}}{}_+$$
$$= V_c(T^*, T, K) + \underbrace{\left(PV_{T^*,T}(K) - F^P_{T^*,T}(S)\right)}_{\text{For simplicity, assume:}}{}_+$$

For simplicity, assume:
- no dividends
- cont-comp, risk-free interest rate (r)

$$= V_c(T^*, T, K) + \underbrace{\left(\underbrace{Ke^{-r(T-T^*)}}_{K^*} - S(T^*)\right)_+}_{\text{PAYOFF of a put w/ strike } Ke^{-r(T-T^*)} \text{ and exercise date } T^*}$$

PAYOFF of a put w/ strike $Ke^{-r(T-T^*)}$ and exercise date T^*

$\Rightarrow$ A replicating portfolio for our chooser option:
- Long call w/ strike K and exercise date T
- Long put w/ strike $Ke^{-r(T-T^*)}$ and exercise date T^*

$$\Rightarrow \quad V_{CH}(0, T^*, T, K) = V_c(0, T, K) + V_p\left(0, T^*, Ke^{-r(T-T^*)}\right)$$

SAMPLE MFE

25. Consider a chooser option (also known as an as-you-like-it option) on a nondividend-paying stock. At time 1, its holder will choose whether it becomes a European call option or a European put option, each of which will expire at time 3 with a strike price of \$100.

$T^* = 1$

$T = 3$

$K = 100$

The chooser option price is \$20 at time $t = 0$. $\qquad Y_{CH}(0) = 20$

The stock price is \$95 at time $t = 0$. Let $C(T)$ denote the price of a European call option at time $t = 0$ on the stock expiring at time T, $T > 0$, with a strike price of \$100.

$S(0) = 95$

You are given:

(i) The risk-free interest rate is 0. $\qquad r = 0$

(ii) $C(1) = \$4$. $\qquad V_c(0,1,100) = 4$

Determine $C(3)$. $\qquad V_c(0,3,100) = ?$

$r = 0$

$$\underbrace{V_{CH}(0)}_{20} = \underbrace{V_c(0,3,100)}_{?} + \underbrace{V_p(0,1,100)}_{\parallel}$$

Put-call Parity

$$V_c(0,1,100) + 100 - 95$$

(A) \$ 9

(B) \$11

(C) \$13

(D) \$15

(E) \$17

answer :

$$V_c(0,3,100) = 20 - 4 - 5 = 11 \Rightarrow B.$$

SAMPLE MFE

3. An insurance company sells single premium deferred annuity contracts with return linked to a stock index, the time-t value of one unit of which is denoted by $S(t)$. The contracts offer a minimum guarantee return rate of $g\%$. At time 0, a single premium of amount π is paid by the policyholder, and $\pi \times y\%$ is deducted by the insurance company. Thus, at the contract maturity date, T, the insurance company will pay the policyholder

$$\pi \times (1 - y\%) \times \text{Max}[S(T)/S(0), (1 + g\%)^T].$$

You are given the following information:

(i) The contract will mature in one year.

(ii) The minimum guarantee rate of return, $g\%$, is 3%.

(iii) Dividends are incorporated in the stock index. That is, the stock index is constructed with all stock dividends reinvested.

(iv) $S(0) = 100$.

(v) The price of a one-year European put option, with strike price of \$103, on the stock index is \$15.21.

Determine $y\%$, so that the insurance company does not make or lose money on this contract.