UNIVERSITY OF TEXAS AT AUSTIN

Lecture 12

Gap options. ... An example of Exoctic options.

12.1. **Gap calls.** A European *gap call option* is a derivative security on an underlying asset (with price denoted by $\mathbf{S} = \{S(t), t \geq 0\}$) which given:

- an exercise date T;
- a **strike** price K_s;
- a **trigger** price K_t

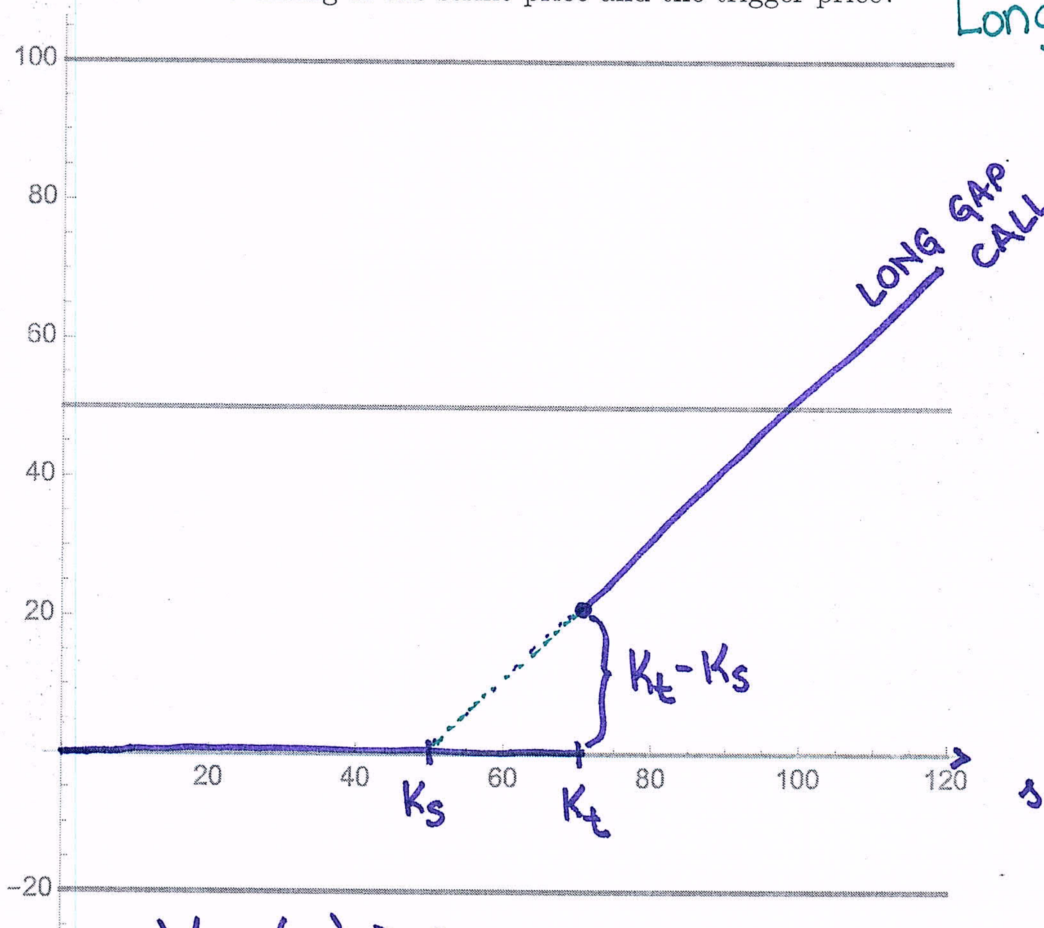
provides the payoff

$$V_{GC}(T) = (S(T) - K_s)\mathbb{I}_{[S(T) \geq K_t]}$$

to its owner.

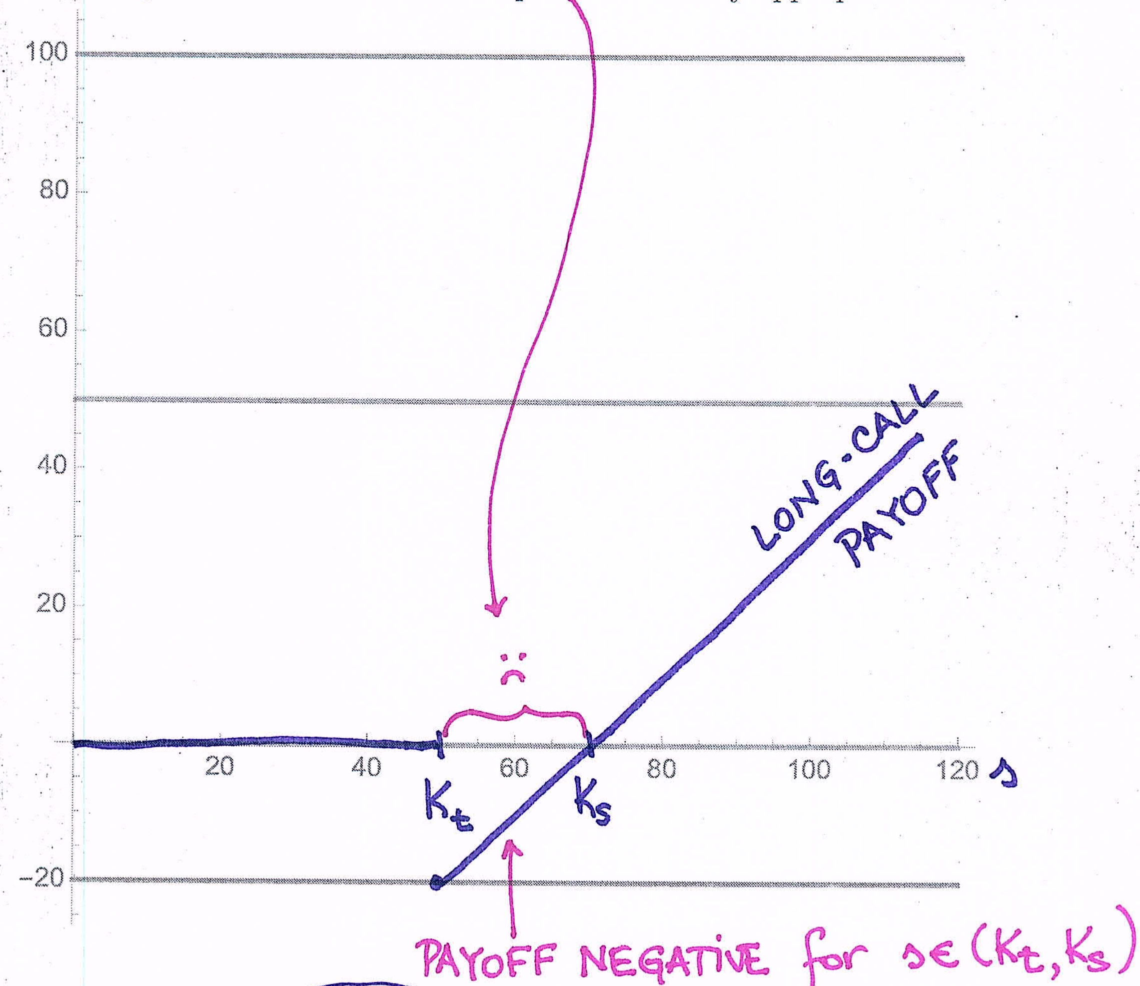
Problem 12.1. Consider a gap call option with $K_s \leq K_t$.

- Draw its payoff curve.
- Is a long gap call a long or a short position with respect to the underlying asset for the above ordering of the strike price and the trigger price? Long !



- $V_{GC}(0) \geq 0$

Problem 12.2. Consider a gap call option with $K_t < K_s$.
- Draw its payoff curve.
- Do you think that the word "option" is entirely appropriate in this case?



$$V_{GC}(T) = (S(T) - K_s)\mathbb{I}_{[S(T) \geq K_t]} = V_{AC}(T) - K_s V_{CC}(T)$$

$$V_{CC}(T) = \mathbb{I}_{[S(T) \geq K_t]}$$

$$V_{AC}(T) = S(T)\mathbb{I}_{[S(T) \geq K_t]}$$

$\Downarrow$

- one LONG asset call w/ trigger price K_t
- K_s SHORT cash calls w/ trigger price

Problem 12.3. Create a $\boxed{\text{replicating portfolio}}$ for the gap call option consisting of cash-or-nothing call options and asset-or-nothing call options.

12.2. **Gap puts.** A European *gap put option* is a derivative security on an underlying asset (with price denoted by $\mathbf{S} = \{S(t), t \geq 0\}$) which given:

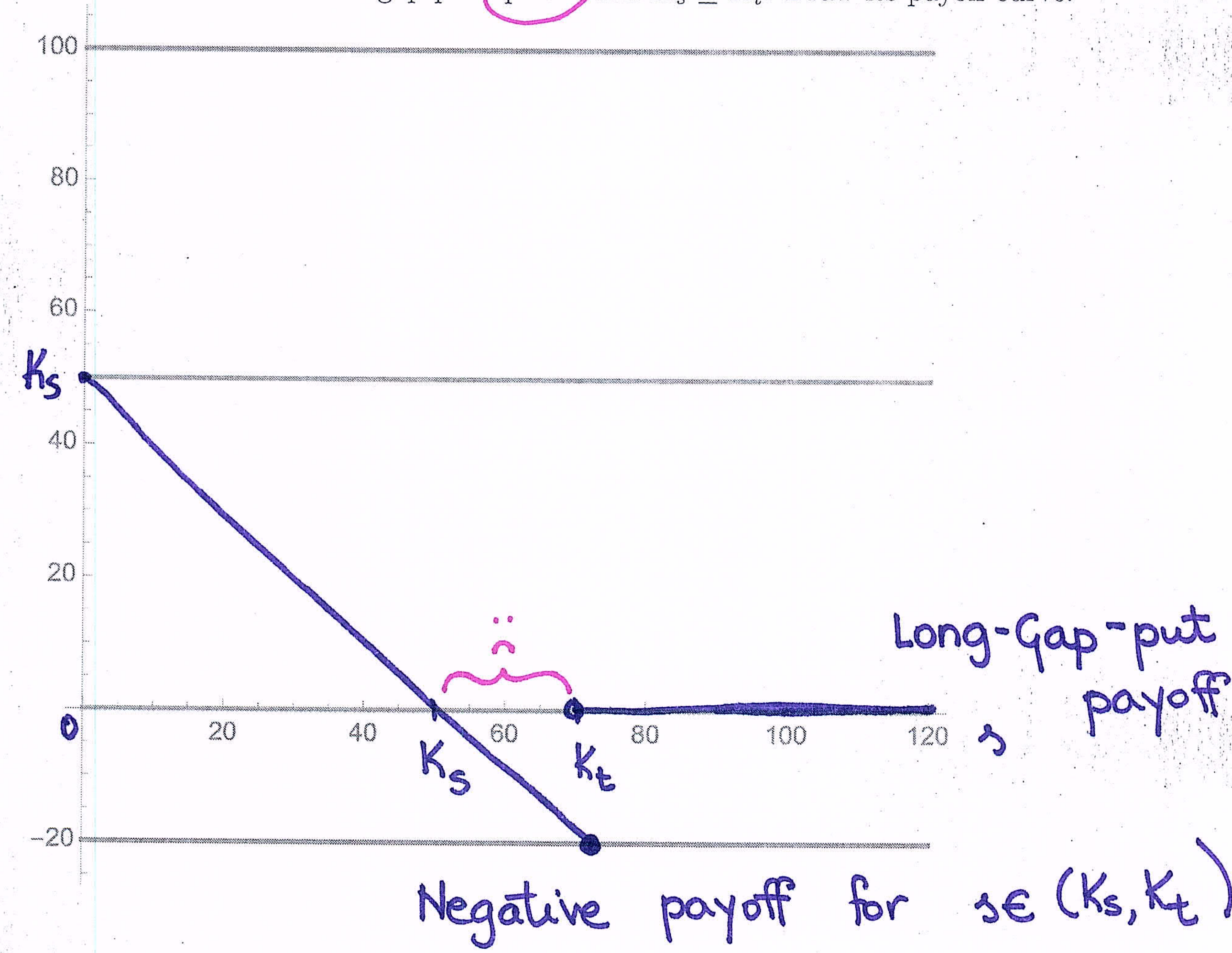
- an exercise date T;
- a **strike** price K_s;
- a **trigger** price K_t

provides the payoff

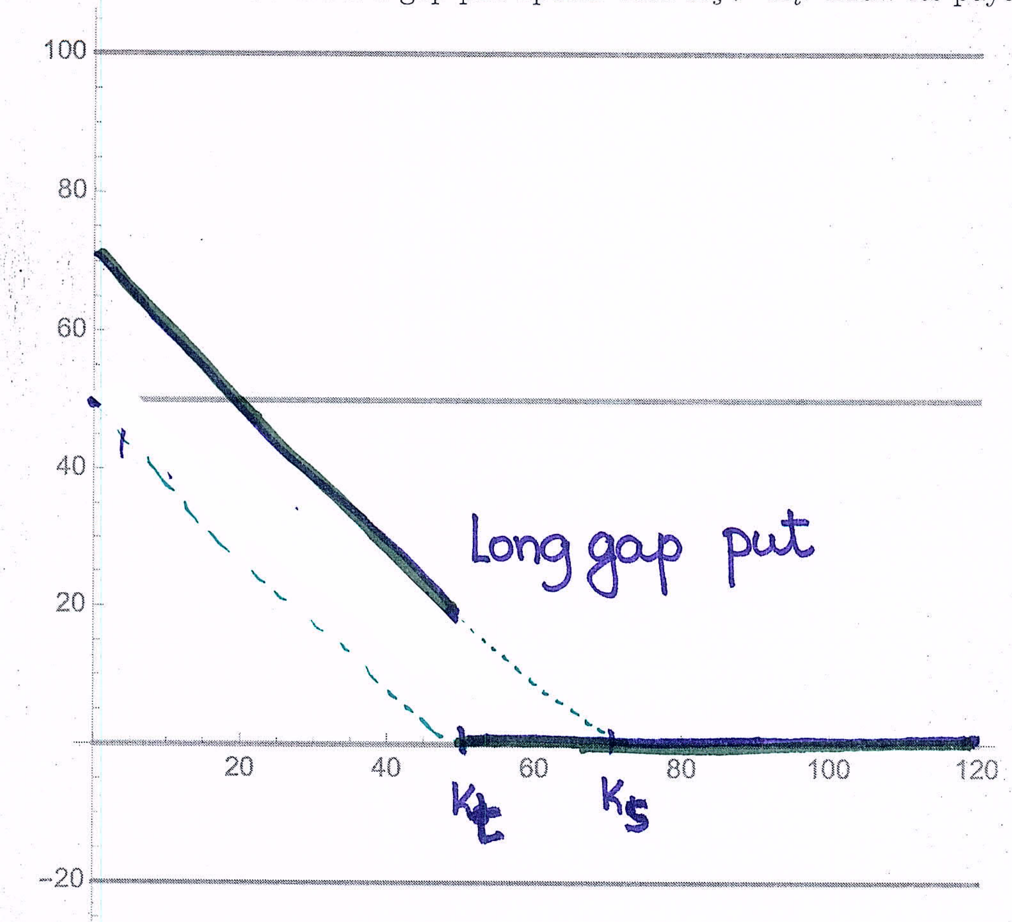
$$V_{GP}(T) = (K_s - S(T))\mathbb{I}_{[S(T)<K_t]}$$

to its owner.

Problem 12.4. Consider a gap put option with $K_s \leq K_t$. Draw its payoff curve.



Problem 12.5. Consider a gap put option with $K_s > K_t$. Draw its payoff curve.



$$V_{GP}(T) = (K_s - S(T))\mathbb{I}_{[S(T)<K_t]} = K_s \cdot V_{CP}(T) - V_{AP}(T)$$

$$\Downarrow$$

$$\left.\begin{array}{l} V_{CP}(T) = \mathbb{I}_{[S(T)<K_t]} \\ V_{AP}(T) = S(T)\mathbb{I}_{[S(T)<K_t]} \end{array}\right\}$$

Long

- K_s — cashputs w/ trigger K_t
- one <u>short</u> asset put w/ trigger K_t

Problem 12.6. Create a replicating portfolio for the gap put option consisting of cash-or-nothing put options and asset-or-nothing put options.

12.3. **Put-call parity for gap options.**

Problem 12.7. Consider the following portfolio:

- one **long** gap call option with trigger price K_t and the strike price K_s,
- one **short** otherwise identical gap put option.

(i) What is the initial cost of the above portfolio expressed in terms of the price of the gap call $V_{GC}(0)$ and the price of the gap put $V_{GP}(0)$?

(ii) What is the payoff of the above portfolio?

(iii) Based on your answers to the above two questions, what is **put-call parity** for gap options?

- Initial cost : $V_{GC}(0) - V_{GP}(0)$

- PAYOFF : $\underline{V_{GC}(T) - V_{GP}(T)} =$

$$= (S(T) - K_s)\mathbb{I}_{[S(T) \geq K_t]} - (K_s - S(T))\mathbb{I}_{[S(T) < K_t]}$$

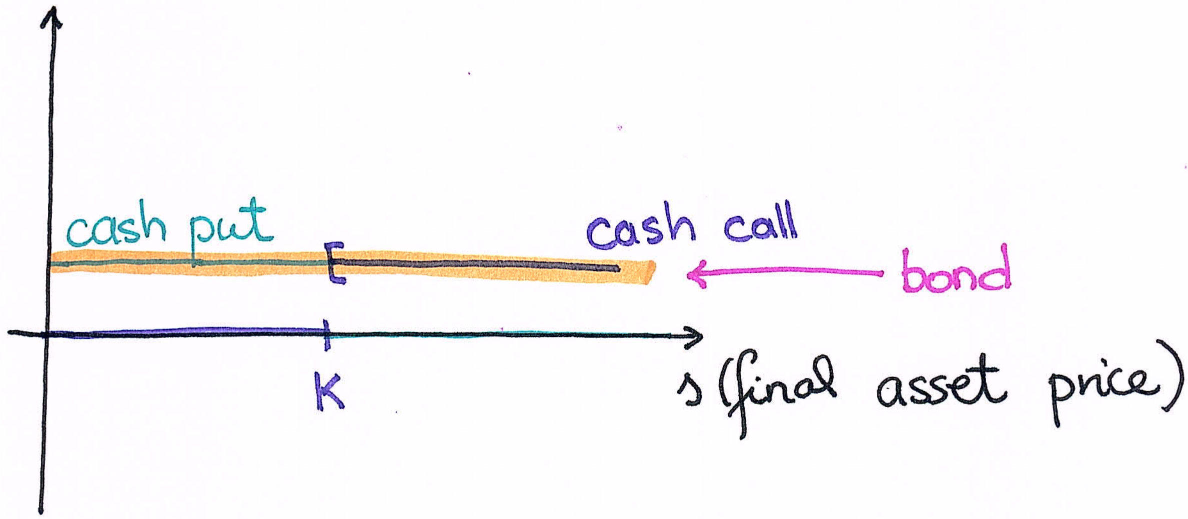
$$= \underbrace{\underline{S(T) - K_s}}_{\text{the trigger price is gone ....}}$$

- $$\boxed{V_{GC}(0) - V_{GP}(0) = F^P_{0,T}(S) - PV_{0,T}(K_s)}$$

Put-call parity.

Problem.

$$V_{CC}(0) + V_{CP}(0) = ?$$



a bond (zero-coupon) redeemable @ time-T for \$1 can be replicated. $\begin{cases} \cdot \text{ long cash call} \\ + \\ \cdot \text{ long cash put} \end{cases}$

$$\Rightarrow \quad V_{CC}(0) + V_{CP}(0) = PV_{0,T}(1) = e^{-rT}$$

Problem.

$$V_{AC}(0) + V_{AP}(0) = \cancel{S} = F^{P}_{0,T}(S)$$

draw the payoff curve :)