

Name:

M339D=M389D Introduction to Actuarial Financial Mathematics
 University of Texas at Austin
Solution: In-Term Exam I
 Instructor: Milica Čudina

Notes: This is a closed book and closed notes exam. The maximal score on this exam is 75 points.

Time: 50 minutes

TRUE/FALSE

	TRUE	FALSE	MULTIPLE CHOICE					
1.2 (2)			1.12 (5)	a	b	c	d	e
1.3 (2)			1.13 (5)	a	b	c	d	e
1.4 (2)			1.14 (5)	a	b	c	d	e
1.5 (2)			1.15 (5)	a	b	c	d	e
1.6 (2)			1.16 (5)	a	b	c	d	e
1.7 (2)			1.17 (5)	a	b	c	d	e
1.8 (2)			1.18 (5)	a	b	c	d	e
1.9 (2)								
1.10 (2)								

FOR THE GRADER'S USE ONLY:

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1.1. **DEFINITIONS.**

Problem 1.1. (10 points) Write the definition of an **arbitrage portfolio**.

1.2. **TRUE/FALSE QUESTIONS.** *Please, circle the correct answer on the front page of this exam.*

Problem 1.2. (2 pts)

A cap consists of a long call option and a long asset. *True or false?*

Solution: FALSE

A cap consists of a long call option and a short asset.

Problem 1.3. (2 points)

A collar is a long position with respect to the underlying asset. *True or false?*

Solution: FALSE

Problem 1.4. (2 points)

Assume that the continuously compounded, risk-free interest rate is strictly positive. The forward price of a non-dividend-paying stock is always strictly increasing with respect to the delivery date. *True or false?*

Solution: TRUE

Problem 1.5. (2 points) A portfolio consisting of a long forward contract and a bond can replicate a long prepaid forward contract. *True or false?*

Solution: TRUE

Problem 1.6. (2 pts) A non-dividend-paying stock sells today for \$100 per share. The yearly effective interest rate is 0.21. Then, $F_{0,1/2}(S) > 110$. *True or false?*

Solution: FALSE

In fact, $F_{0,1/2}(S) = 100(1.21)^{1/2} = 100 \times 1.1 = 110$.

Problem 1.7. (2 points) The profit diagram and the payoff diagram for long positions in a forward contract are identical. *True or false?*

Solution: TRUE

Problem 1.8. If the stock pays discrete dividends, there is a comparative advantage to an outright purchase of the stock as compared to the prepaid forward contract on that stock. *True or false?*

Solution: FALSE

The prepaid forward price includes the “compensation” for forfeited dividends payments.

Problem 1.9. (2 points) *Source: Sample IFM (Derivatives – Intro) Problem #24.* Derivative securities can be used as a means of hedging. *True or false?*

Solution: TRUE

Problem 1.10. In our usual notation, the difference between the **profit** of a long forward contract and a long investment in one unit of the non-dividend-paying underlying asset equals the forward price. *True or false?*

Solution: FALSE

$$S(T) - F - S(T) + S(0)e^{rT} = S(0)e^{rT} - F.$$

Of course, if the available forward price is the no-arbitrage forward price, the profits are equal.

1.3. **FREE-RESPONSE PROBLEMS.**

Please, record your final numerical answers in the boxes at the bottom of every page with a free-response problem. Single-word or one-number answers without a justification will receive zero credit.

Problem 1.11. (15 points) You produce tiramisu cakes. You plan to sell 1,000 cakes in a month. Your revenue will be $\$10,000 - S(1)$, where $S(1)$ denote the price of the amount of belgian chocolate required to dust the 1,000 cakes.

Assume that the continuously compounded annual risk-free interest rate equals 6%.

Your hedge consists of the following two components:

- (1) one **long** one-month, \$9,000-strike call option on the amount of chocolate you need; it's premium is $V_C(0) = \$60.00$,
- (2) one **written** one-month, \$8,500-strike put option on the amount of chocolate you need; it's premium is $V_P(0) = \$200.00$.

Calculate the range of profits of the hedged portoffio.

Solution: The hedged portfolio consists of the following components:

- (1) **revenue** from the tiramisu cake sales,
- (2) one **long** one-year, \$9,000-strike call option on the chocolate whose premium was \$60.00,
- (3) one **written** one-year, \$8,500-strike put option on the chocolate whose premium was \$200.00.

The initial cost for this portfolio is the cost of hedging (all other accumulated production costs are incorporated in the revenue expression $10,000 - S(1)$). Their future value is

$$(60 - 200)e^{0.005} \approx -140.70.$$

As usual, the negative initial cost signifies an initial influx of money for the principal character (in this case, you: the producer of tiramisu cakes).

The range of the payoffs is the range of values of

$$10000 - s + (s - 9000)_+ - (8500 - s)_+ = \begin{cases} 10000 - s - 8500 + s = 1500, & \text{for } s < 8500, \\ 10000 - s, & \text{for } 8500 < s < 9000, \\ 10000 - s + s - 9000 = 1000, & \text{for } 9000 < s. \end{cases}$$

So, the range of payoffs is $[1000, 1500]$. Taking into account the future value of the initial cost of heging, we get that the range of profits equals $[1140.70, 1640.70]$.

1.4. MULTIPLE CHOICE QUESTIONS.

Please, circle the correct answer on the front page of this exam.

Problem 1.12. (5 points) You are tasked with buying oranges in the market in grove A, transporting the oranges to a juice factory in the market B, and selling the oranges to the juice factory in the market B. You want to hedge. Which of the following would be a satisfactory hedge?

- (a) Long a call in market A and long a put in market B
- (b) Short a call in market A and long a put in market B
- (c) Long a call in market A and short a put in market B
- (d) Short a call in market A and short a put in market B
- (e) None of the above.

Solution: (a)

Problem 1.13. (5 points) A portfolio consists of the following:

- one **long** 40–strike put option,
- one **long** 50–strike call options,
- two **short** 60–strike put options.

All of the options are European, with the same underlying asset and the same exercise date. What is the portfolio's **payoff** if the final price of the underlying asset equals \$55?

- (a) –10
- (b) –5
- (c) 15
- (d) 25
- (e) None of the above.

Solution: (b)

$$(40 - 55)_+ + (55 - 50)_+ - 2(60 - 55)_+ = 0 + 5 - 10 = -5.$$

Problem 1.14. (5 points) *Source: Sample MFE (Intro) Problem #20.*

The current price of a stock is 200, and the continuously compounded risk-free interest rate is 4%. A dividend will be paid every quarter for the next 3 years, with the first dividend occurring 3 months from now. The amount of the first dividend is 1.50, but each subsequent dividend will be 1% higher than the one previously paid. Calculate the fair price of a 3-year forward contract on this stock.

- (a) \$200.41
- (b) \$205.41
- (c) \$210.41
- (d) \$215.41
- (e) None of the above.

Solution: (b)

Problem 1.15. (5 points) In which of the following option positions is the investor exposed to an unlimited loss?

- (a) Long a put option
- (b) Short a put option
- (c) Long a call option
- (d) Short a call option
- (e) None of the above

Solution: (d)

Just draw the payoff diagrams to convince yourselves.

Problem 1.16. (5 points) A soy-bean farmer shorts forward contracts on soy in an amount matching his crop volume and with delivery at harvest time. Then, he is considered:

- (a) an arbitrageur.
- (b) a broker.
- (c) a speculator.
- (d) a hedger.
- (e) None of the above.

Solution: (d)

Problem 1.17. (5 points) The initial price of a continuous-dividend-paying market index equals \$1,000. The dividend yield equals 0.03.

An investor simultaneously purchases one unit of the index and a one-year, 975-strike European put option on the index for a premium of \$10.

In one year, the spot price of the index is observed to be \$950.

Given that the continuously compounded risk-free interest rate equals 0.03, what is the profit of the investor's portfolio? *Caveat: Be careful with continuous reinvestment of dividend in the index.*

- (a) 36.83 loss
- (b) 61.82 loss
- (c) 61.82 gain
- (d) 36.83 gain
- (e) None of the above.

Solution: (a)

In our usual notation,

$$e^{\delta T}S(T) + (K - S(T))_+ - (S(0) + V_P(0))e^{rT} = e^{0.03} \times 950 + 25 - 1010e^{0.03} = 25 - 60e^{0.03} = -36.83$$

Problem 1.18. (5 points) The “Very tasty goat cheese Co” sells artisan goat cheese at \$10 per oz. They need to buy 200 gallons of goat milk in six months to make 200 oz of their specialty fall-equinox cheese. Non-goat milk aggregate costs total \$500. They decide to buy six-month, \$5-strike call options on gallons of goat milk for 0.50 per call option.

The continuously compounded risk-free interest rate equals 0.04.

In six months, the price of goat milk equals \$6 per gallon. What is the profit of the company's hedged position?

- (a) 395.92
- (b) 397.98
- (c) 400
- (d) 897.98
- (e) None of the above.

Solution: (b)

$$200 \times 10 - 200 \times 5 - 500 - 200 \times 0.50e^{0.02} = 397.98$$