UNIVERSITY OF TEXAS AT AUSTIN

Problem set 2

Final exam prep. More problems were requested.

Problem 2.1. (2 points) In the setting of the one-period binomial model, denote by i the **effective** interest rate **per period**. Let u denote the "up factor" and let d denote the "down factor" in the stock-price model. If

$$d < u \leq 1 + i$$

then there certainly is no possibility for arbitrage.

Solution: FALSE

Problem 2.2. (2 points) A compound call on a put option costs at most as much as the underlying put option itself.

Solution: TRUE

Problem 2.3. (2 points) A bull spread is a long position with respect to the underlying asset.

Solution: TRUE

Problem 2.4. (2 points) An agent is **only** allowed to long a forward contract if he/she is willing to take physical delivery of the underlying asset.

Solution: FALSE
It is possible to have *cash settlement* on the delivery date if the forward contract stipulates so.

Problem 2.5. (2 points) Stock options may be used for employee compensation.

Solution: TRUE

Problem 2.6. (2 points) You are using a binomial asset-pricing model to model the evolution of the price of a particular stock. Then, the Δ in the replicating portfolio of a single call option on that stock never exceeds 1.

Solution: TRUE
The call's Δ will always be between 0 and 1.

Problem 2.7. (2 points) A long collar is used to hedge an inherent long position (a position of the *producer* of goods, e.g.).

Solution: TRUE

Problem 2.8. (2 points) Consider a one-year, \$55-strike European call option and a one-year, \$45-strike European put option on the same underlying asset.

You observe that the time-0 stock price equals \$40 while the time-1 stock price equals \$50. Then, both of the options are out-of-the-money at expiration.

Solution: TRUE

Problem 2.9. (2 points) Naked writing is the practice of buying options without taking an offsetting position in the underlying asset.

Solution: FALSE

Problem 2.10. Consider a non-dividend-paying stock whose current price is \$100 per share. Its volatility is given to be 0.25. You model the evolution of the stock price over the following year using a two-period forward binomial tree.

The continuously compounded risk-free interest rate is 0.04.

Consider a \$110-strike, one-year **down-and-in** put option with a barrier of \$90 on the above stock. What is the price of this option consistent with the above stock-price model?

(a) About \$10.23
(b) About \$11.55
(c) About \$11.78
(d) About \$11.90
(e) None of the above.

Solution: (a)
The up and down factors in the above forward binomial tree are

$$u = e^{0.02+0.25/\sqrt{2}} = 1.2089, \quad d = 0.8610.$$

Of course, in the exam, you would not necessarily populate the entire stock-price tree, since you would want to work efficiently and only consider the nodes you need for pricing.

The option is knocked-in only if the stock price goes down in the first step. So, the payoff of the option will be

$$\begin{aligned} V_{du} &= 5.92, && \text{if the path } \textit{down-up} \text{ is taken,} \\ V_{dd} &= 35.87, && \text{if the path } \textit{down-down} \text{ is taken,} \\ V_{uu} = V_{ud} &= 0, && \text{otherwise.} \end{aligned}$$

The risk-neutral probability of a single step up in the tree equals

$$p^* = \frac{1}{1+e^{0.3/\sqrt{2}}} = 0.4577.$$

So, the option price is

$$V(0) = e^{-rT}[(1-p^*)^2 V_{dd} + p^*(1-p^*)V_{du}] = 10.23.$$

Problem 2.11. A customer buys a six-month at-the-money put on an index when the market price of the index is 50. The premium for the put is 2.

The continuously compounded risk-free interest rate equals 0.06.

The price of the index at expiration is modeled as follows

$$\begin{aligned} &45, && \text{with probability } 0.6, \\ &50, && \text{with probability } 0.3, \\ &55, && \text{with probability } 0.1. \end{aligned}$$

What is the expected value of the profit of the long put?

(a) \$0.53
(b) \$1.03
(c) \$1.12
(d) \$2.50
(e) None of the above.

Solution: (e)

$$(50-45)*0.6 - 2e^{0.03} = 0.939.$$

Problem 2.12. (5 points) The future value in one year of the total aggregate costs of manufacturing a widget is \$100. You will sell a widget in one year at its market price of $S(1)$.

Assume that the continuously compounded risk-free interest rate equals 5%.

You purchase a one-year, \$120-strike put on one widget for a premium of \$7. You sell some of the potential gain by writing a one-year, \$150-strike call on one widget for a \$3 premium.

What is the **range** of the profit of your total hedged porfolio?

(a) $[14.75, 44.75]$
(b) $[15.79, 45.79]$
(c) $[16.20, 46.20]$
(d) $[120, 150]$
(e) None of the above.

Solution: (b)
The payoff, written as a piecewise function, is

$$v(s) = \begin{cases} K_P, & \text{for } 0 \leq s \leq K_P \\ s, & \text{for } K_P \leq s \leq K_C \\ K_C, & \text{for } K_C \leq s \end{cases}$$

where K_P denotes the strike price for the put while K_C denotes the call's strike price. So, the range of the payoff function is $[120, 150]$.

The future value of the total cost of both production and hedging is

$$100 + (7-3)e^{-0.05} = 104.21.$$

So, the range of the profit equals $[15.79, 45.79]$.

Problem 2.13. (5 points)

Today's price of a market index paying continuous dividends equals \$80. The projected dividend yield is given to be 0.02.

The continuously compounded risk-free interest rate equals 0.04.

You notice that the price of a six-month, \$75-strike European call options equals \$8.51. What is the price of the otherwise identical European put option?

(a) About \$2.02
(b) About \$2.15
(c) About \$2.82
(d) About \$3.50
(e) None of the above.

Solution: (c)
By put-call parity, we have that

$$V_P(0) = V_C(0) - F_{0,T}^P(S) + PV_{0,T}(K) = 2.8175.$$

Problem 2.14. (2 points) For any given positive strike price K_S, there exists a trigger price K_T which makes the price of a gap put equal to the price of a vanilla put with the same strike.

Solution: TRUE
The gap is "closed" for $K_S = K_T$.

Problem 2.15. (2 points) An *American straddle* is a position whose payoff function equals $v(s) = |s - K|$ for some strike price K. More precisely, if T denotes the **expiration date** of the straddle, the owner of the straddle can at any time $t \in [0, T]$ decide to exercise the straddle and get the payoff equal to $|S(t) - K|$.

Then, the simultaneous purchase of an American call with exercise date T and strike K and the otherwise identical American put forms a replicating portfolio for the American straddle.

Solution: FALSE

Problem 2.16. (2 points) Derivative securities can reduce the risk of both the buyer and the writer of the security.

Solution: TRUE
Forward contracts are an example of this situation.

Problem 2.17. The current exchange rate is \$0.80 per Swiss franc. The continuously compounded risk-free interest rate for the US dollar is 4%, while the continuously compounded risk-free interest rate for the Swiss franc equals 6%.

A franc-denominated European call option on \$100 is available in the market at a premium of 12.70 Swiss francs. Its exercise date is in one year, and its strike price is 115 Swiss francs.

What is the price of the otherwise identical put option?

(a) About 0.90 Swiss francs.
(b) About 5.47 Swiss francs.
(c) About 7.60 Swiss francs.
(d) About 44.14 Swiss francs.
(e) None of the above.

Solution: (a)
Put-call parity for currency options gives us

$$V_P(0) = 12.70 + 115e^{-0.06} - 125e^{-0.04} = 0.904$$

Problem 2.18. A long straddle position:

(a) is a speculation on the stock's volatility.
(b) can be replicated with a long call and a short put with the same strike, underlying asset and exercise date.
(c) is less expensive than the corresponding strangle.
(d) is equivalent to a short butterfly spread.
(e) None of the above.

Solution: (a)

Problem 2.19. Consider a non-dividend paying stock whose current price is \$95 per share. You model the evolution of this stock price over the following year using a one-period binomial tree under the assumption that the stock price can be either \$120, or \$75 in one year.

The continuously compounded risk-free interest rate is 0.06.

Consider a \$100-strike, one-year European **straddle** on the above stock. What is the straddle's price consistent with the above stock-price model?

(a) About \$10
(b) About \$10.83
(c) About \$15.45
(d) About \$20.84
(e) None of the above.

Solution: (d)
The risk-neutral probability of an up movement is

$$p^* = \frac{95e^{0.06} - 75}{120 - 75} = 0.575.$$

So, the price of our straddle is

$$V_C(0) = e^{-0.06}[0.575 \times (120 - 100) + (1 - 0.575) * (100 - 75)] = 20.8366.$$

Problem 2.20. The current price of a non-dividend-paying stock is $80 per share. You observe that the price of a three-month, at-the-money European put option on this stock equals $2.50.

The continuously compounded risk-free interest rate is 0.08.

Find the premium of the European three-month, at-the-money call option on the same underlying asset.

(a) About $3.08
(b) About $4.08
(c) About $4.75
(d) About $5.46
(e) None of the above.

Solution: (b)
Recall that the price of an American call on a non-dividend-paying stock equals the price of the otherwise identical European call option. So, put-call parity yields

$$V_C(0) = V_P(0) - Ke^{-rT} + S(0) = 2.50 - 80(e^{-0.02} - 1) = 4.0841.$$

Problem 2.21. (2 points)

Bermuda-style options are always at most as valuable as otherwise identical **American-style** options. *True or false?*

Solution: TRUE