Early exercise. Moneyness. Forward binomial trees (multi period).

Please, provide complete solution(s) to the following problem(s):

Problem 6.1. (5 points) Source: Sample MFE Problem Set (Intro); Problem #46.

Determine which of the following statements about options is true:

(A) Naked writing is the practice of buying options without taking an offsetting position in the underlying asset.

(B) A covered call involves taking a long position in an asset together with a written call on the same asset.

(C) An American style option can only be exercised during specified periods, but not for the entire life of the option.

(D) A Bermudan style option allows the buyer the right to exercise at any time during the life of the option.

(E) An in-the-money option is one which would have a positive profit if exercised immediately.

Solution: (B)

Problem 6.2. (10 points)

The current stock price of a continuous-dividend-paying stock is 100 per share. Its volatility is 0.25 and its dividend yield is 0.01.

Assume that the continuously compounded risk-free interest rate equals 0.04.

(a) (3 points) Construct a two-period forward binomial tree for the above stock modeling the stock price evolution over the following half-year.

(b) (2 points) Consider a half-year, $95-strike call option on the stock. What is the moneyness of this option at the up and down nodes?

(c) (5 points) Assume that the above option is European. What is its price?

Solution:

(a) This is a forward binomial tree, so the up and down factors are

\[ u = e^{(r - \delta)h + \sigma \sqrt{h}} = e^{(0.04 - 0.01)(0.25) + 0.25(0.5)} = 1.1417, \]

\[ d = e^{(r - \delta)h - \sigma \sqrt{h}} = e^{(0.04 - 0.01)(0.25) - 0.25(0.5)} = 0.8891. \]

So, we have

\[ S_u = 114.17, \quad S_{uu} = 130.34, \]

\[ S_{ud} = 101.51, \]

\[ S_d = 88.91, \quad S_{dd} = 79.06. \]

(b) The option is in-the-money at the up node, and out-of-the-money at the down node.

(c) The risk-neutral probability of the stock price going up in a single step is

\[ p^* = \frac{1}{1 + e^{\sigma \sqrt{h}}} = \frac{1}{1 + e^{0.25\sqrt{0.5}}} = 0.4688. \]

The price of our call option is

\[ V_C(0) = e^{-0.04/2} \left[ 35.34(0.4688)^2 + 2(0.4688)(1 - 0.4688)(6.51) \right] = 10.7921. \]