UNIVERSITY OF TEXAS AT AUSTIN

Quiz #12
Properties of option prices. Spreads.

Provide your **final answer only** to the following problem(s):

Problem 12.1. (5 points) We are given the following European-call prices for options on the same underlying asset:

$$\begin{array}{ll} \$50\text{-strike} & \$11 \\ \$55\text{-strike} & \$6 \\ \$60\text{-strike} & \$4 \end{array}$$

Assume that the continuously compounded interest rate is strictly positive. Which of the following portfolios would exploit an arbitrage opportunity stemming from the above stock prices?
   (a) The call bull spread only.
   (b) The call bear spread only.
   (c) Both the call bull and the call bear spread.
   (d) Neither the call bull or call bear spread, but there is an arbitrage opportunity.
   (e) There is no apparent arbitrage opportunity.

Solution: (b)

Problem 12.2. (2 points) The payoff of the call bull spread is equal to the payoff of the put bull spread. *True or false?*

Solution: FALSE
It's the profits that are equal.

Problem 12.3. (2 points) A butterfly spread can be constructed in this way:
          Buy a 90 call, sell a 100 put, sell a 100 call, buy a 110 put.

True or false?

Solution: TRUE

Problem 12.4. (2 pts) In our usual notation, we **always** have that
$$V_C(t) > S(t) - Ke^{-r(T-t)}$$
for every $t \in [0, T]$ regardless of whether the stock pays dividends or not. *True or false?*

Solution: FALSE

Problem 12.5. (5 points) Consider three European put options on the same stock with the same exercise date. The put premium for the $32-$strike option is 2.50 and the put premium for the $37-$strike option is 6.50. What can you say about the $40-$strike put option?
   (a) Its highest possible premium is $8.90.
   (b) Its lowest possible premium is $8.90.
   (c) Its highest possible premium is $10.50
   (d) Its lowest possible premium is $10.50.
   (e) None of the above.

Solution: (b)
To satisfy the convexity condition for put prices with respect to the strike, with x denoting the lowest possible $40-$strike put price, we get

$$\frac{3}{8} \times 2.5 + \frac{5}{8}x = 6.50 \quad \Rightarrow \quad x = 8.9.$$