Name:

M339D=M389D Introduction to Actuarial Financial Mathematics
University of Texas at Austin
The Prerequisite In-Term Exam
Instructor: Milica Čudina

Notes: This is a closed book and closed notes exam. The maximal score on this exam is 100 points.
Time: 50 minutes

TRUE/FALSE

0.1 (2)	TRUE	FALSE
0.2 (2)	TRUE	FALSE
0.3 (2)	TRUE	FALSE
0.4 (2)	TRUE	FALSE
0.5 (2)	TRUE	FALSE

MULTIPLE CHOICE

0.16 (5)	a	b	c	d	e
0.17 (5)	a	b	c	d	e
0.18 (5)	a	b	c	d	e
0.19 (5)	a	b	c	d	e

FOR THE GRADER'S USE ONLY:

T/F	F.R.	M.C.	Σ	

0.1. **TRUE/FALSE QUESTIONS.** *Please, circle the correct answer on the front page of this exam.*

Problem 0.1. (2 pts) 23% of 61 is greater than 61% of 23. *True or false?*

Problem 0.2. (2 pts)
We define the minimum of two values in the usual way, i.e.,

$$\min(x, y) = \begin{cases} x & \text{if } x \leq y \\ y & \text{if } x \geq y \end{cases}$$

Then, for every x and y we have that

$$\min(x, y) = \min(x - y, 0) + y$$

True or false?

Problem 0.3. (2 pts)
We define the maximum of two values in the usual way, i.e.,

$$\max(x, y) = \begin{cases} x & \text{if } x \geq y \\ y & \text{if } x \leq y \end{cases}$$

Then, for every x and y we have that

$$-\max(x, y) = \max(x - y, 0) - x$$

True or false?

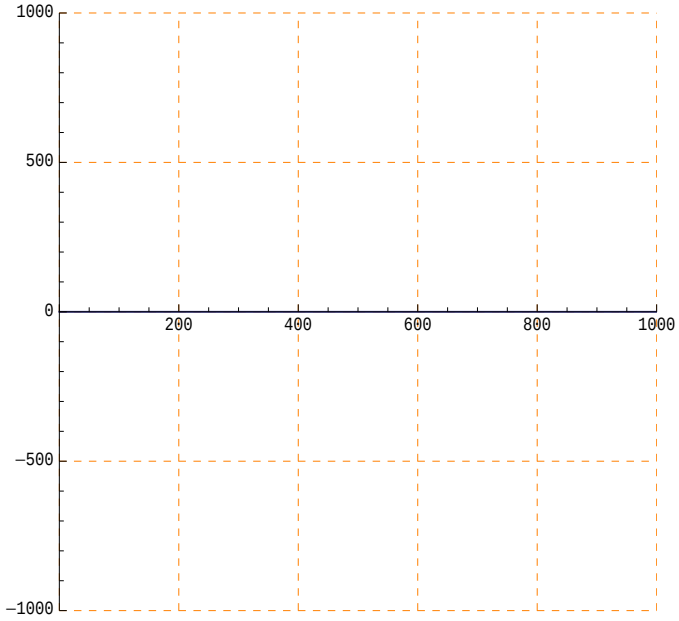
Problem 0.4. (2 pts) Two dice are rolled, the probability that the sum of the upturned faces equals 7 is 1/6. *True or false?*

Problem 0.5. (2 pts) Two dice are rolled, the probability that the maximum (and **not** necessarily a strict maximum) of the upturned faces is achieved on the second die equals 1/2. *True or false?*

0.2. **FREE-RESPONSE PROBLEMS.**

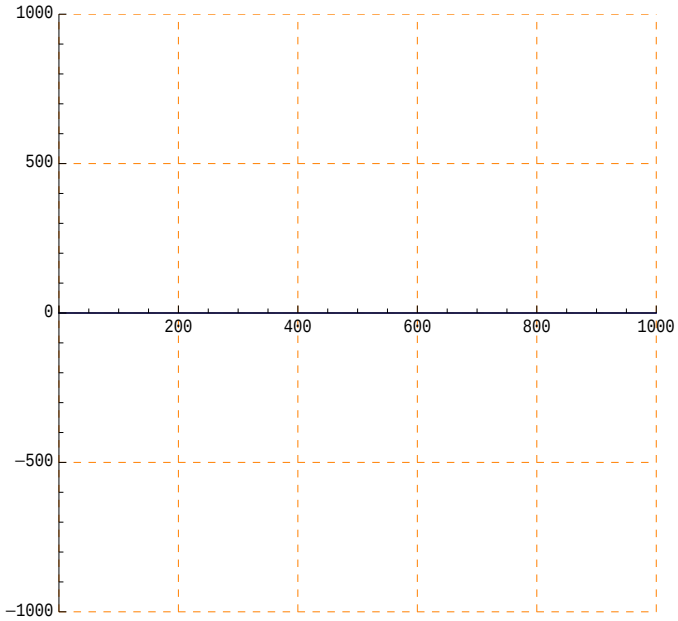
Problem 0.6. (5 points) Draw the graph of the following function in the coordinate system provided below:

$$f(x) = \begin{cases} 0 & \text{for } x < 600 \\ x - 600 & \text{for } x \geq 600 \end{cases}$$



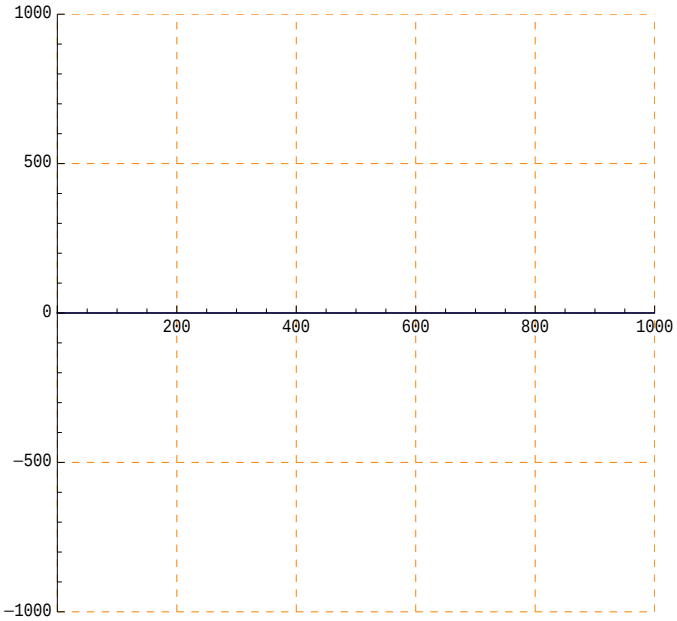
Problem 0.7. (5 points) Draw the graph of the following function in the coordinate system provided below:

$$f(x) = \begin{cases} x & \text{for } x < 500 \\ 2x - 500 & \text{for } x \geq 500 \end{cases}$$



Problem 0.8. (6 points) Draw the graph of the following function in the coordinate system provided below:

$$f(x) = \begin{cases} |x - 500| & \text{for } x < 800 \\ x & \text{for } x \geq 800 \end{cases}$$



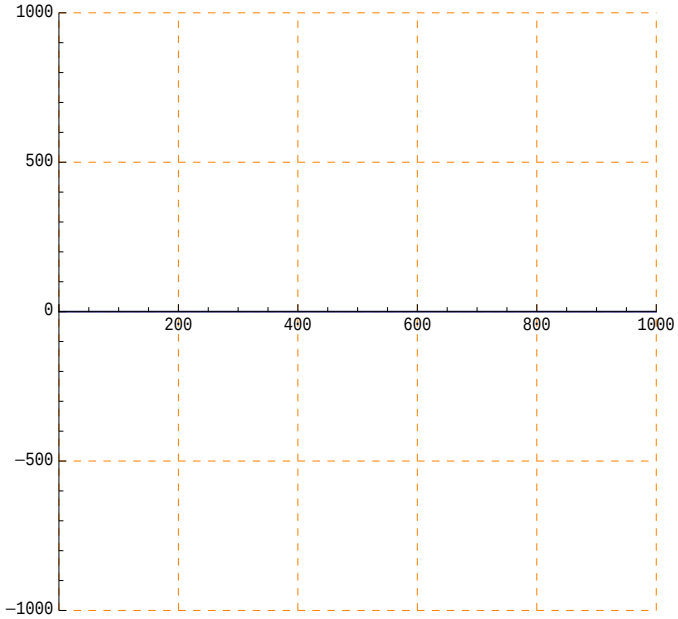
Problem 0.9. (6 points) Let the function f be given by

$$f(x) = \begin{cases} x - 300 & \text{for } x \geq 300 \\ 0 & \text{otherwise} \end{cases}$$

Draw the graph of the function g defined as

$$g(x) = f(x) - 50$$

in the coordinate system provided.



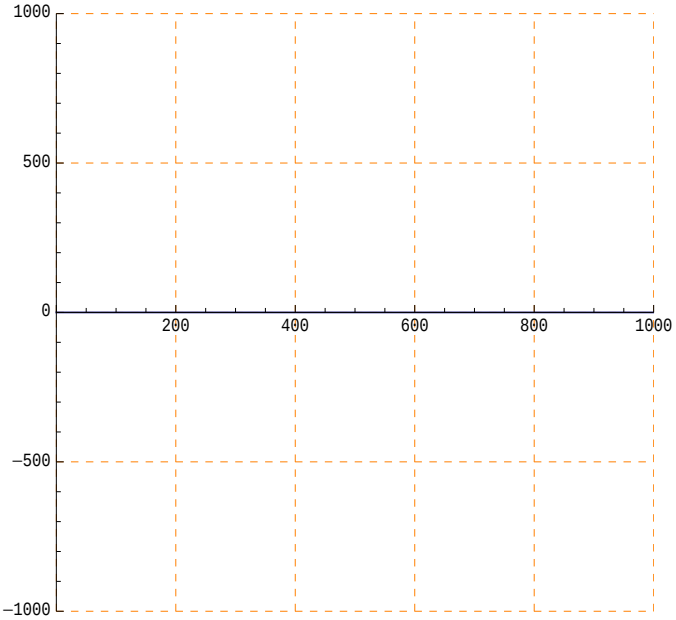
Problem 0.10. (8 points) Let the function f be defined as

$$f(x) = x$$

Let the function g be defined as

$$g(x) = \begin{cases} 0 & \text{for } x < 500 \\ x - 500 & \text{for } x \geq 500 \end{cases}$$

Draw the graph of the function $f + g$ in the coordinate system provided below:



Problem 0.11. (5 points) Assume that a loan is taken out and that the loan repayment scheme is such that every installment is exactly equal to the amount of interest charged on the loan since the last payment time; in other words, every payment consists only of the interest due. What is the expression for the **outstanding loan balance** immediately after the k^{th} payment is made?

If you believe that you have insufficient information to answer this question, just write down: "Insufficient info."

Problem 0.12. (5 pts) Find the total amount of interest that would be paid on a \$1,000 loan over a 10−year period, if the effective interest rate is 0.09 per annum under the following repayment method:

Interest is paid each year as it is accrued and the principal is repaid at the end of the loan term.

Problem 0.13. (10 points)

Emmanuel entered an extra special kind of game with his friend Fischer. First, they toss a fair coin. If the coin comes up heads, Emmanuel gives $\$5,000$ to Fischer. If the coin comes up tails, Fischer gives $\$2,000$ to Emmanuel. Then, regardless of the outcome of the first cointoss, they toss the same fair coin again. If it comes up heads, Emmanuel gives Fischer \$4,000. If the coin comes up tails, Fischer gives \$3,000 to Emmanuel. What is the expected cashflow, i.e., what is the expected amount of money that changes hands and who gives it to whom?

Problem 0.14. (15 points)

Harry plays a simple lottery in which the winnings are distributed as follows:

- \$0 with probability 0.1,
- \$10 with probability 0.3,
- \$20 with probability 0.6.

(i) (5 points) What is the expected value of the amount Harry wins?

(ii) (10 points) Unfortunately, there is a catch to the lottery Harry plays. It turns out that Harry has to pay a fee to collect his winnings. If the actual amount he wins is smaller than \$15, then the fee is defined to equal the amount that Harry won – thus, he walks away with nothing. If the actual amount he wins is larger than \$15, then he pays the \$15-fee and pockets the remainder. What is the expected value of the net amount Harry collects?

Problem 0.15. (5 points) Twelve people enter the elevator on the ground floor of a ten-story buiilding and start riding up. By the time the elevator reaches the top floor, all the people have exited the elevator. What is the probability that at least two people exited the elevator on the same floor?

0.3. MULTIPLE CHOICE QUESTIONS.

Please, circle the correct answer on the front page of this exam.

Problem 0.16. (5 pts) UT Shuttles arrive at a certain stop during a certain time period at fifteen minute intervals starting at the full hour. A student arrives at the stop at a random time that is uniformly distributed between the full hour and the half-hour. What is the probability that the student waits for a bus for less then 5 minutes?

(a) 1/15
(b) 1/10
(c) 1/5
(d) 1/3
(e) None of the above

Problem 0.17. (5 pts) A pair of dice is thrown. Find the probability that the sum of the outcomes is 10 or greater **if** a 5 appears on at least one of the dice.

(a) 1/6
(b) 3/11
(c) 1/3
(d) 1/2
(e) None of the above

Problem 0.18. (5 pts) Let $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ be two functions given by

$$f(x) = x - 10$$

and

$$g(x) = \begin{cases} x & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$

Then, $g(f(3))$ equals ...

(a) -13
(b) -10
(c) -7
(d) 0
(e) None of the above

Problem 0.19. (5 pts) Roger deposits \$100 into an account at time 0.
For the following three years, he does not make any subsequent withdrawals or deposits and the account earns at a constant continuously compounded, risk-free interest rate r.

After 15 years and 6 months, the balance in his account equals \$133. Then,

(a) $0 \leq r < 0.0150$
(b) $0.0150 \leq r < 0.0250$
(c) $0.0250 \leq r < 0.0550$
(d) $0.0550 \leq r < 0.0650$
(e) None of the above