Name:

M339D=M389D Introduction to Actuarial Financial Mathematics
University of Texas at Austin
Sample In-Term Exam II
Instructor: Milica Čudina

Notes: This is a closed book and closed notes exam. The maximal score on this exam is 100 points.
Time: 50 minutes

TRUE/FALSE

1 (2)	TRUE	FALSE
2 (2)	TRUE	FALSE
3 (2)	TRUE	FALSE
4 (2)	TRUE	FALSE
5 (2)	TRUE	FALSE

MULTIPLE CHOICE

1 (5)	a	b	c	d	e
2 (5)	a	b	c	d	e
3 (5)	a	b	c	d	e
4 (5)	a	b	c	d	e
5 (5)	a	b	c	d	e
6 (5)	a	b	c	d	e
7 (5)	a	b	c	d	e
8 (5)	a	b	c	d	e

FREE-RESPONSE PROBLEMS

Problem 2.1. (?? points) An investor wants to hold 200 euros two years from today. The spot exchange rate is \$1.31 per euro. If the euro denominated annual interest rate is 3.0% what is the price of a currency prepaid forward?

Problem 2.2. (?? points) There are two European options on the same stock with the same time to expiration. The 90-strike call costs \$20 and the 100-strike call costs \$8.

Is there an arbitrage opportunity due to the above call prices?

Propose an arbitrage portfolio (if you concluded that it exists) and **verify** that your proposed portfolio is indeed an arbitrage portfolio.

Problem 2.3. (?? points)

Consider a continuous-dividend-paying stock whose current price is \$40 and whose dividend yield is 0.02. The price of stock in three months is modeled using a one-period binomial tree.

The continuoulsy compounded, risk-free interest rate is 0.06.

According to the above stock-price model, the replicating portfolio of an at-the-money, three-month European call option consists of:

- 0.6 shares of stock, and
- borrowing \$20 at the risk-free interest rate.

What is the risk-free portion of the replicating portfolio for the otherwise identical put option?

2.1. **MULTIPLE CHOICE QUESTIONS.** *Please, circle the correct answer on the front page of this exam.*

Problem 2.4. You construct an asymmetric butterfly spread using the following three types of European options on the same asset and with the same exercise date:

- a \$50-strike call,
- a \$60-strike call,
- a \$65-strike call.

You are told that there is exactly **one** short \$60-strike call in the asymmetric butterfly spread. What is the maximal payoff of the above butterfly spread?

(a) 0
(b) 10/3
(c) 5
(d) The payoff is not bounded from above.
(e) None of the above.

Problem 2.5. The following two one-year European put options on the same asset are available in the market:

- a \$50-strike put with the premium of \$5,
- a \$55-strike put with the premium of \$10.

The continuously compounded, risk-free interest rate is 0.04.

Which of the following positions certainly exploits the arbitrage opportunity caused by the above put premia?

(a) Put bull spread.
(b) Put bear spread.
(c) Both of the above positions.
(d) There is no arbitrage opportunity.
(e) None of the above.

Problem 2.6. (5 points) Which one of the following positions always has an infinite upward potential in the sense that the payoff diverges to positive infinity as the argument s (standing for the final stock price) tends to positive infinity?

(a) A long call option.
(b) A long ratio spread.
(c) A long bull spread.
(d) A long butterfly spread.
(e) None of the above.

Problem 2.7. The current price of stock **S** is \$50. Stock **S** is scheduled to pay a \$3-dividend in two months.

The current price of stock **Q** is \$60. Stock **Q** is scheduled to pay dividends continuously with the dividend yield 0.03.

A six-month European exchange call option with underlying asset **S** and the strike asset **Q** is sold for \$2.75.

The continuoulsy compounded, risk-free interest rate is given to be 0.04.

What is the price of the six-month European exchange put option with underlying asset **S** and the strike asset **Q**?

(a) About \$8.58
(b) About \$9.04
(c) About \$12.75
(d) About \$14.54
(e) None of the above.

Problem 2.8. A long strangle position...

(a) is equivalent to a short ratio spread.
(b) can be replicated with a short call and a long put with the same strike, underlying asset and exercise date.
(c) is always strictly more expensive than the straddle on the same underlying asset and with the same exercise date.
(d) is a speculation on the stock's volatility.
(e) None of the above.

Problem 2.9. You are given that the price of:

- a \$50-strike, one-year European call equals \$8,
- a \$65-strike, one-year European call equals \$2.

Both options have the same underlying asset. What is the maximal price of a \$56-strike, one-year European call such that there is no arbitrage in our market model?

(a) \$4.40
(b) \$5
(c) \$5.60
(d) \$6.02
(e) None of the above.

Problem 2.10. An investor bought a six-month, $(70, 80)$-put **bull** spread on an index. The \$70-strike, six-month put is currently valued at \$1, while the \$80-strike, six-month put is currently valued at \$8.

Assume that the continuoulsy compounded, risk-free interest rate equals 0.02.

What is the **break-even** final index price for the above put bull spread?

(a) \$62.86
(b) \$71.84
(c) \$72.93
(d) \$73.23
(e) None of the above.

Problem 2.11. Consider a continuous-dividend-paying stock with the current price of \$45 and dividend yield 0.02.

The continuoulsy compounded, risk-free interest rate is 0.04.

Consider a pair of six-month, \$50-strike, \$45-trigger gap options. The gap call sells for \$1.70. What is the price of the gap put?

(a) \$5.17
(b) \$6.16
(c) \$7.27
(d) \$7.41
(e) None of the above.

Problem 2.12. Consider a continuous-dividend-paying stock with the current price of \$50 and dividend yield 0.02.

The continuoulsy compounded, risk-free interest rate is 0.05.

You are using a one-period binomial tree to model the stock price at the end of the next quarter. You assume that the stock price can either increase by 0.04 or decrease by 0.02. What is the risk-neutral probability associated with this tree?

(a) 0.3675
(b) 0.4588
(c) 0.5430
(d) 0.8409
(e) None of the above.

2.2. TRUE/FALSE QUESTIONS.

Problem 2.13. The expiration date of a futures option cannot exceed the delivery date of the underlying futures contract.

Problem 2.14. (2 points) The payoff curve of a **call bear** spread is never positive.

Problem 2.15. (2 points) The payoff of a gap put option is always nonnegative regardless of the choice of the trigger and the strike. *True or false?*

Problem 2.16. (2 pts) Suppose that the European options with the same maturity and the same underlying assets have the following prices:

(1) a 50−strike call costs $9;
(2) a 55−strike call costs $10;

Then, one should *acquire* a **call bear spread** to exploit the arbitrage since some of the monotonicity conditions for no-arbitrage are violated by the above premiums.

Problem 2.17. (2 pts) Consider a European gap put option. Then, the premium of this option is *decreasing* with respect to the strike price (the trigger price is being held constant!).

Problem 2.18. (2 points)

The following is a replicating portfolio for a *ratio spread*:

Long a two-year European call and write a three-year European call with the same strike price and the same underlying asset.

True or false?

Problem 2.19. (2 points) A **long** strangle has a non-negative payoff function.
True or false?

Problem 2.20. (2 points) Consider a gap option whose trigger price is equal to its strike price. Then, the premium for this option is the same as that for a vanilla European option with the same strike, the same exercise date and the same underlying asset. *True or false?*

Problem 2.21. Suppose that prices of European calls on the same asset and with the same exercise date for varying strike prices are given by the following table

Strike	80	100	105
Call premium	22	9	5

Then, one can use a butterfly spread to exploit the violations of the no-arbitrage conditions exhibited by the prices in the above table. *True or false?*

Problem 2.22. The strike price at which the European call and the otherwise identical European put have the same premiums is the forward price for delivery of the underlying on the exercise date of the two options. *True or false?*

Problem 2.23. (2 points) Exchange options are options where the underlying asset is an exchange rate.

Problem 2.24. (2 points)

In the setting of the binomial asset-pricing model, let d and u denote the up and down factors, respectively. Moreover, let r denote the continuously compounded, risk-free interest rate. Let h denote the length of a single period in our model.

Then, if,

$$e^{\delta h}d < e^{rh} < e^{\delta h}u$$

then there is no possibility for arbitrage. *True or false?*