The Payoff Curve

... the graph of the payoff function.

Example. Fully-leveraged stock purchase

\[ \text{PAYOFF} = e^{ST} \cdot S(T) - S(0)e^{rT} \]

\[ v^*(s) = e^{ST} \cdot S(T) - S(0)e^{rT} \]

represents the final stock price

Def'n. For any financial portfolio the payoff function is the final cashflow (for the investor's perspective) represented as a function of the final stock price.
A fully-leveraged purchase

A valid payoff curve

$\psi(s)$

Payoff curve

$-S(0) e^{rt}$

$S$ (final asset price)
The profit curve

... the graph of the function

\[ u(s) = \text{FV}_s(I_C) \]

Payoff Function


The payoff: \(-SCT\)  \(\Rightarrow\)  \(u(s) = -s\)

The profit: \(-SCT - (-S(0)e^{rT}) = -S(T) + S(0)e^{rT}\)

\(\Rightarrow\)  \(-s + S(0)e^{rT}\)
Example: Outright purchase of cont-div. stock
\[ V(s) = e^{sT} \cdot s \]

Profit and Payoff diagram:
- Payoff: \[ S(0)e^{rT} \]
- Profit: \[ e^{rT} \cdot x = S(0)e^{rT} \]

Break Even:
\[ x^* = S(0)e^{(r-s)T} \]
Monotonicity

\( \mathbb{R} \ldots \text{the real line} \)

Def'n. We say that a function \( f : \mathcal{D}_f \subseteq \mathbb{R} \rightarrow \mathbb{R} \)

is

- **increasing** / non-decreasing
  
  \[ x_1 < x_2 \Rightarrow f(x_1) \leq f(x_2) \]

- **decreasing** / non-increasing
  
  \[ x_1 < x_2 \Rightarrow f(x_1) \geq f(x_2) \]
Example.