European call options (Review)

Q: Does the writer of the option need to own the underlying asset in order to be allowed to write the option?

No.

- If he/she does own the asset, this is called **COVERED OPTION WRITING**.
- If the writer does **NOT** have a position in the underlying, this is called **NAKED OPTION WRITING**.

⇒ • Written/Short Call option} **COVERED CALL**
  • Long Asset
Expected call payoff

Asset (e.g., stocks, gold, copper, corn, ...) has \( S(T) \) ... asset price on the exercise date modeled as:

\[
S(T) \sim \begin{cases} 
  x_1 & p_1 \\
  x_2 & p_2 \\
  \vdots & \vdots \\
  x_k & p_k \\
  \vdots & \vdots \\
  x_m & p_m 
\end{cases}
\]

Consider a European call on this asset w/ strike \( K \) and the exercise date \( T \).
Q: What is the expected payoff of this call option?

Option payoffs:

\[
(S(T) - K)_+ \sim \begin{cases} 
  (x_1 - K)_+ & p_1 \\
  (x_2 - K)_+ & p_2 \\
  \vdots & \vdots \\
  (x_k - K)_+ & p_k \\
  \vdots & \vdots \\
  (x_m - K)_+ & p_m 
\end{cases}
\]

\[
E \left[ (S(T) - K)_+ \right] = \sum_{k=1}^{m} p_k (x_k - K)_+
\]
European put options

... give the owner/holder of the put option the RIGHT, but **not** an obligation, to SELL a unit of the underlying for a predetermined strike price \( K \) on the exercise date \( T \). The **writer** of the put is obligated to do as the option owner chooses.

**At** \( t = T \) observe \( S(T) \), i.e., the final asset price.

\[
\text{IF} \quad S(T) \leq K \quad \text{THEN} \quad \text{(EXERCISE)}
\]

\[
\text{IF} \quad S(T) > K \quad \text{THEN} \quad \text{not}
\]

\[\Rightarrow \text{The payoff:} \quad V_p(T) = (K - S(T))^+\]

\[\Rightarrow \text{The payoff function:} \quad v_p(s) = (K - s)^+\]

\[
\text{PAYOFF} \quad \rightarrow K \quad \text{Long-put Payoff}
\]

\[
\text{upper bound} \quad \rightarrow \text{Long-put Profit} \quad \text{break even} \quad s^* = K - FV_{0,T}(V_p(0))
\]

\[
\text{\textbullet Decreasing} \Rightarrow \text{A SHORT Position w/ respect to the underlying} \quad \text{\textbullet Nonnegative} \Rightarrow \text{An initial premium}
\]

\[
S \text{ (final asset price)} \quad \text{\textbullet } (K - s)^+ - FV_{0,T}(V_p(0)) = 0
\]

\[
K - s^* - FV_{0,T}(V_p(0)) = 0
\]
Example: A producer of goods hedging using a European put

\[
\begin{align*}
\text{PAYOFF} & \quad \text{HEDGED} \quad \text{unhedged} \\
K & \quad \text{hedge (Long Put)} \\
K & \quad s \text{ (final asset price)}
\end{align*}
\]

Same shape as a call's payoff.

FLOOR: \[\begin{cases}
\text{Long asset} \\
\text{Long put}
\end{cases}\]

COVERED PUT: \[\begin{cases}
\text{Short/written put} \\
\text{Short-sell the asset}
\end{cases}\]