Name:

M339D=M389D Introduction to Actuarial Financial Mathematics
University of Texas at Austin
In-Term Exam II
Instructor: Milica Čudina

Notes: This is a closed book and closed notes exam. The maximal score on this exam is 65 points.
Time: 50 minutes

TRUE/FALSE

2.1 (2)	TRUE	FALSE
2.2 (2)	TRUE	FALSE
2.3 (2)	TRUE	FALSE
2.4 (2)	TRUE	FALSE
2.5 (2)	TRUE	FALSE
2.6 (2)	TRUE	FALSE
2.7 (2)	TRUE	FALSE
2.8 (2)	TRUE	FALSE

MULTIPLE CHOICE

2.10 (5)	a	b	c	d	e
2.11 (5)	a	b	c	d	e
2.12 (5)	a	b	c	d	e
2.13 (5)	a	b	c	d	e
2.14 (5)	a	b	c	d	e
2.15 (5)	a	b	c	d	e
2.16 (5)	a	b	c	d	e
2.17 (5)	a	b	c	d	e

FOR GRADER'S USE ONLY:

T/F	2.9	M.C.	Σ

2.1. **TRUE/FALSE QUESTIONS.** *Please, circle the correct answer on the front page of this exam.*

Problem 2.1. (2 points) The payoff of the call bull spread is equal to the payoff of the put bull spread. *True or false?*

Solution: FALSE
It's the profits that are equal.

Problem 2.2. (2 points)
You long a $(90, 100, 110)-$butterfly spread with one long \$90-strike call. Then, you short one \$110-strike European call with the same exercise date and underlying asset. The portfolio you end up with is equivalent to a ratio spread. *True or false?*

Solution: TRUE

Problem 2.3. (2 pts) In our usual notation, we **always** have that

$$V_C(t) > S(t) - Ke^{-r(T-t)}$$

for every $t \in [0, T]$ regardless of whether the stock pays dividends or not. *True or false?*

Solution: FALSE

Problem 2.4. (2 points) The chooser option with the exercise date T and with the strike K is worth at least as much as a vanilla call with the same underlying, strike and exercise date. *True or false?*

Solution: TRUE

Problem 2.5. (2 points) The payoff curve of a call bear spread is never positive. *True or false?*

Solution: TRUE

Problem 2.6. (2 points) A **box** spread is a replicating portfolio for a zero-coupon bond. *True or false?*

Solution: TRUE

Problem 2.7. (2 points) Exchange options are options where the underlying asset is an exchange rate. *True or false?*

Solution: FALSE

Problem 2.8. (2 points) A **straddle** has a nonnegative profit function. *True or false?*

Solution: FALSE

2.2. **FREE-RESPONSE PROBLEMS.**

Problem 2.9. (10 points) There are two European options on the same stock with the same time to expiration. The 90-strike put costs \$8 and the 100-strike put costs \$20.

The risk free interest rate is 0.05.

Is there an arbitrage opportunity due to the above put prices? Propose an arbitrage portfolio (if you concluded that it exists) and **verify** that your proposed portfolio is indeed an arbitrage portfolio.

Solution:
Diagnosis. The given put prices are obviously increasing with respect to the strike price, so let us check if the amount of increase is appropriately bounded from above. We have

$$V_P(100) - V_P(90) = 20 - 8 > 10 \geq PV_{0,T}(100 - 90). \tag{2.1}$$

We conclude that there exists an arbitrage oportunity to be exploited since the upper bound on the amount of increase in put prices is violated.
Arbitrage-portfolio proposal. As we can see from inequality (2.1), the 100-strike put is relatively expensive when compared to the 90-strike put. So, we propose the following arbitrage portfolio:

- a **long** 90-strike put,
- a **short** 100-strike put.

The above position is commonly referred to as a **(put) bull spread**.
Verification. The initial cost of the above portfolio is

$$V_P(90) - V_P(100) = 8 - 20 = -12 < -PV_{0,T}(10).$$

The payoff curve of the above put bull spread is bounded from below by -10.

So, the profit of the above put bull spread is strictly greater than

$$-10 + FV_{0,T}(-12) \geq 2 > 0.$$

We have shown that the portfolio we proposed has a strictly positive profit in all states of the world. This is an even stronger requirement than the one from the definition of an arbitrage portfolio!

2.3. MULTIPLE CHOICE QUESTIONS.

Please, circle the correct answer on the front page of this exam.

Problem 2.10. (5 points) An investor buys a two-year (\$800, \$900)-strangle on gold. The price of gold two years from now is modeled using the following distribution:

$$\begin{aligned}
&\$750, \quad \text{with probability } 0.45,\\
&\$850, \quad \text{with probability } 0.4,\\
&\$925, \quad \text{with probability } 0.15.
\end{aligned}$$

What is the investor's expected payoff?

(a) About \$23.25
(b) About \$25.00
(c) About \$26.25
(d) About \$37.50
(e) None of the above.

Solution: (c)

$$50 \times 0.45 + 25 \times 0.15 = 26.25.$$

Problem 2.11. (5 points) Consider the ratio spread consisting of:

- five long \$40-strike, one-year calls on **S**,
- seven short \$60-strike, one-year calls on **S**.

You model the stock price at time−1 using the following model

$$S(1) \sim \begin{cases} \$35, & \text{with probability } 0.15\\ \$45, & \text{with probability } 0.25\\ \$55, & \text{with probability } 0.35\\ \$65, & \text{with probability } 0.25 \end{cases}$$

What is the expected payoff of the ratio spread above?

(a) 45
(b) 50
(c) 55
(d) 60
(e) None of the above

Solution: (c)

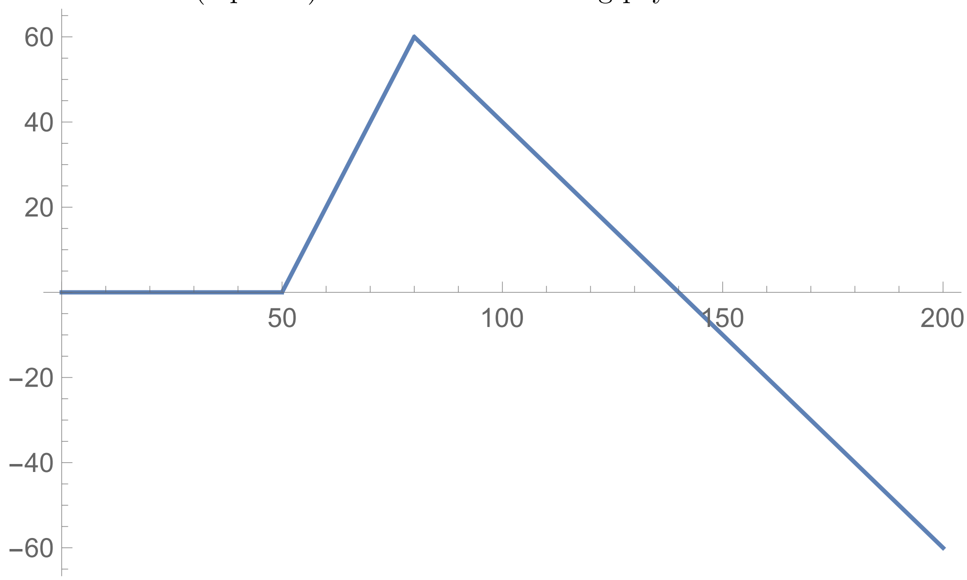
$$5(45-40)(0.25) + 5(55-40)(0.35) + 5(65-40)(0.25) - 7(65-60)(0.25) = 55$$

Problem 2.12. Consider a non-dividend-paying stock. Which of the following portfolios has the same profit as a $(40, 50)-$bull spread?

(a) A **long** $(40, 50)-$collar and a short stock.
(b) A **short** $(40, 50)-$collar and a long stock.
(c) A **long** $40-$strike call, a **written** $50-$strike put, and a long stock.
(d) A **long** $40-$strike call, a **written** $50-$strike put, and a short stock.
(e) None of the above.

Solution: (d)

Problem 2.13. (5 points) Consider the following payoff curve:



Which of the following positions has the above payoff?

(a) A long butterfly spread.
(b) A short butterfly spread.
(c) A ratio spread.
(d) A short straddle.
(e) None of the above.

Solution: (c)

Problem 2.14. (5 points) Consider a European call option and a European put option on a non-dividend paying stock $\mathbf{S}$. You are given the following information:

(1) $r = 0.04$;
(2) The current price of the call option $V_C(0)$ is by 0.15 greater than the current price of the put option $V_P(0)$;
(3) Both the put and the call expire in 4 years.
(4) The put and the call have the same strikes equal to 70.

Find the spot price $S(0)$ of the underying asset.

(a) 48.90
(b) 59.80
(c) 69.70
(d) 79.60
(e) None of the above.

Solution: (b)
Using put-call parity, we get

$$0.15 = V_C(0) - V_P(0) = S(0) - Ke^{-rT} = S(0) - 70e^{-0.04\times 4} \quad \Rightarrow \quad S(0) = 0.15 + 70e^{-0.16} = 59.80.$$

Problem 2.15. Let the continuously-compounded, risk-free interest rate be equal to 0.04. Consider a one-period binomial tree with every period of length one year used to model the stock price of a stock whose current price is \$80 per share. The stock pays dividends continuously with a dividend yield of 0.02. In the model, it is assumed that the stock price can either go up by \$5 or down by \$4.

You use the binomial tree to construct a replicating portfolio for a $(78, 82)-$strangle on the above stock. What is the stock investment in the replicating portfolio?

(a) Long 0.1089 shares.
(b) Long 0.33 shares.
(c) Short 0.1089 shares.
(d) Short 0.33 shares.
(e) None of the above.

Solution: (a)
The two possible stock prices are $S_u = 85$ and $S_d = 76$. So, the possible payoffs of the strangle are $V_u = 3$ and $V_d = 2$. The Δ of the strangle, thus, equals

$$\Delta = e^{-\delta h}\frac{V_u - V_d}{S_u - S_d} = e^{-0.02}\frac{3-2}{85-76} = 0.108911. \tag{2.2}$$

Problem 2.16. Let the continuously-compounded, risk-free interest rate be equal to 0.04. Consider a one-period binomial tree with every period of length one year used to model the stock price of a non-dividend-paying stock whose current price is \$50 per share. In the model, it is assumed that the stock price can either go up by 5% or down by 10%.

You use the binomial tree to construct a replicating portfolio for a $(45, 55)-$call bull spread on the above stock. What is the risk-free investment in the replicating portfolio?

(a) Borrow \$45
(b) Borrow \$43.24
(c) Lend \$45

(d) Lend \$43.24
(e) None of the above.

Solution: (b)
The two possible stock prices are $S_u = 52.5$ and $S_d = 45$. So, the possible payoffs of the call bull spread are $V_u = 7.5$ and $V_d = 0$. The risk-free investment B in the replicating portoflio of the call bull spread, thus, equals

$$B = e^{-rh}\frac{uV_d - dV_u}{u-d} = e^{-0.04}\frac{1.05(0) - 0.9(7.5)}{1.05 - 0.9} = -43.2355. \tag{2.3}$$

Problem 2.17. You are given that the price of:

- a \$50-strike, one-year European call equals \$8,
- a \$65-strike, one-year European call equals \$2.

Both options have the same underlying asset. What is the maximal price of a \$56-strike, one-year European call such that there is no arbitrage in our market model?

(a) \$4.40
(b) \$5
(c) \$5.60
(d) \$6.02
(e) None of the above.

Solution: (c)
Using the convexity of call price with respect to the strike, we get the following answer:

$$\frac{3}{5} \times 8 + \frac{2}{5} \times 2 = \frac{24 + 4}{5} = 5.60.$$