

UNIVERSITY OF TEXAS AT AUSTIN

## Lecture 6

Put-call parity: The general case

6.1. **Construction.** So far, we have looked at put-call parity for non-dividend-paying assets. Now, we will use a similar approach to obtain put-call parity for stocks that pay either discrete dividends, or a continuous dividend stream.

Let *Portfolio A* consist of a long European call and a short European put on the same underlying asset  $S$  with the same strike  $K$  and the same exercise date  $T$ . The initial value of this portfolio is

$$V_A(0) = V_C(0) - V_P(0).$$

There are no intermediate cash-flows associated with this portfolio and its payoff at time  $T$  is

$$V_C(T) - V_P(T) = S(T) - K.$$

On the other hand, let *Portfolio B* consist of the following:

- (1) a **long** prepaid forward contract on  $S$  for delivery at time  $T$ ,
- (2) **borrowing** the present value of the strike price to be repaid at time  $T$ .

Then, the initial cost of this portfolio equals:

$$F_{0,T}^P(S) - PV_{0,T}(K).$$

Since there are no intermediate cash-flows associated with this portfolio, either, its payoff at time  $T$  is

$$S(T) - K.$$

Since the above portfolios have the same final payoff, by the no-arbitrage principle, we conclude that their initial values must also be the same. We get the more general version of put-call parity:

$$V_C(0) - V_P(0) = F_{0,T}^P(S) - PV_{0,T}(K).$$

6.2. **Special cases.** Our most common setting is the one with a continuously compounded interest rate  $r$ . In that case the put-call parity reads as

$$V_C(0) - V_P(0) = F_{0,T}^P(S) - Ke^{-rT}.$$

With respect to dividends, these are the three cases we will be looking into:

- non-dividend-paying stocks:

$$V_C(0) - V_P(0) = S(0) - Ke^{-rT}$$

- discrete dividends  $D_i, i = 1, \dots, n$  at times  $0 < t_1 < \dots < t_n \leq T$ :

$$V_C(0) - V_P(0) = S(0) - \sum_{i=1}^n D_i e^{-rt_i} - Ke^{-rT}$$

- continuous dividends at the rate  $\delta$ :

$$V_C(0) - V_P(0) = S(0)e^{-\delta T} - Ke^{-rT}$$

**6.3. MFE Exam Spring 2007: Problem #1.** On April 30, 2007, a common stock is priced at \$52.00. You are given that:

- (1) Dividends in equal amounts are to be paid on June 30, 2007, and on September 30, 2007.
- (2) A European call on the above stock with strike  $K = \$50$  and the exercise date in six months sells for \$4.50.
- (3) A European put on the above stock with strike  $K = \$50$  and the exercise date in six months sells for \$2.45.
- (4) The continuously-compounded risk-free interest rate equals 0.06.

Calculate the amount of each dividend.

*Solution.* In addition to our usual notation, we introduce  $D$  to stand for the amount of each dividend payment. Then, the put-call parity reads as

$$V_C(0) - V_P(0) = S(0) - De^{-rt_1} - De^{-rt_2} - Ke^{-rT}$$

with  $t_1 = 1/6$  and  $t_2 = 5/12$ . Solving for  $D$  above, we get

$$D = \frac{S(0) - Ke^{-rT} - V_C(0) + V_P(0)}{e^{-rt_1} + e^{-rt_2}} = \frac{52 - 50e^{-0.06 \cdot (1/2)} - 4.5 + 2.45}{e^{-0.06 \cdot (1/6)} + e^{-0.06 \cdot (5/12)}} \approx 0.73.$$