M339J: February 10^{th}, 2020.

PROBLEM. Let N_1 and N_2 be independent Poisson random variables w/ $\mathbb{E}[N_1]=1$ and $Var[N_2]=2$. Find $\mathbb{P}[N_1+N_2=4]$.

→: From the STAM tables, we know

$$\mathbb{E}[N_1] =: \lambda_1 = 1$$

and

$$Var[N_2] =: \lambda_2 = 2.$$

Set $N := N_1 + N_2$.

Q: What is the dist'n of N?

→: Since N_1 & N_2 are independent, we have that

$$P_N(z) = P_{N_1}(z) \cdot P_{N_2}(z)$$

From the STAM Tables:

$$P_{N_i}(z) = e^{\lambda_i(z-1)} \quad \text{for } i=1,2.$$

$$\Rightarrow P_N(z) = e^{\lambda_1(z-1)} \cdot e^{\lambda_2(z-1)}$$

$$= e^{(\lambda_1+\lambda_2)(z-1)}$$

We recognize: $N \sim Poisson(\lambda_1+\lambda_2)$

In our problem:

$$N \sim \text{Poisson}(\lambda = 3)$$

$$\Rightarrow \mathbb{P}[N=4] = \frac{e^{-3} \cdot 3^4}{4!} = \frac{e^{-3} \cdot 81}{24} = \frac{27e^{-3}}{8}$$

$$\approx 0.168 \blacksquare$$

172. A policyholder has probability 0.7 of having no claims, 0.2 of having exactly one claim, and 0.1 of having exactly two claims. Claim amounts are uniformly distributed on the interval [0, 60] and are independent. The insurer covers 100% of each claim.

Calculate the probability that the total benefit paid to the policyholder is 48 or less.

(A) 0.320
(B) 0.400
(C) 0.800
(D) 0.892
(E) 0.924

✗ 173. In a given region, the number of tornadoes in a one-week period is modeled by a Poisson distribution with mean 2. The numbers of tornadoes in different weeks are mutually independent.

Calculate the probability that fewer than four tornadoes occur in a three-week period.

(A) 0.13
(B) 0.15
(C) 0.29
(D) 0.43
(E) 0.86

174. An electronic system contains three cooling components that operate independently. The probability of each component's failure is 0.05. The system will overheat if and only if at least two components fail.

Calculate the probability that the system will overheat.

(A) 0.007
(B) 0.045
(C) 0.098
(D) 0.135
(E) 0.143

3.

$\rightarrow$: N ... total # of tornadoes in a three-week period

$N \sim \text{Poisson}(\lambda = 2 \cdot 3 = 6)$

$$\mathbb{P}[N < 4] = p_N(0) + p_N(1) + p_N(2) + p_N(3)$$

$$= e^{-6}\left(\frac{6^0}{0!} + \frac{6^1}{1!} + \frac{6^2}{2!} + \frac{6^3}{3!}\right)$$

$$= e^{-6}(1 + 6 + 18 + 36) = 61e^{-6} \approx 0.1512 \quad \blacksquare$$

Reading Assignment:

Section 3.5: Risk Measures

Example. [SCALING & THE NORMAL DIST'N]

$X \sim N(\text{mean} = \mu_X, \text{var} = \sigma_X^2)$

Consider a constant c; set

$\tilde{X} = c \cdot X$

$\tilde{X} \sim N(\text{mean} = c \cdot \mu_X, \text{var} = c^2 \cdot \sigma_X^2)$

$\uparrow$

$\text{Var}[c \cdot X] = c^2 \cdot \text{Var}[X]$

Example. [SCALING & THE WEIBULL DIST'N]

$X \sim \text{Weibull}(\theta, \tau)$

Let $c > 0$ be a constant.

$\tilde{X} := c \cdot X$

Let's find the cdf of $\tilde{X}$!

$$F_{\tilde{X}}(x) = \mathbb{P}[\tilde{X} \le x] = \mathbb{P}[cX \le x] \qquad (c>0)$$

$$= \mathbb{P}\left[X \le \frac{x}{c}\right] = 1 - e^{-\left(\frac{x/c}{\theta}\right)^{\tau}}$$

$$= 1 - e^{-\left(\frac{x}{c\cdot\theta}\right)^{\tau}}$$

$$\Rightarrow \text{Weibull}(\underbrace{\tilde{\theta} = c\cdot\theta}_{\text{Scale Parameter!}}, \tau)$$