✱ **71.** The time, T, that a manufacturing system is out of operation has cumulative distribution function

$$F(t)=\begin{cases}1-\left(\dfrac{2}{t}\right)^2, & t>2\\ 0, & \text{otherwise.}\end{cases}$$

The resulting cost to the company is $Y=T^2$. Let g be the density function for Y.

Determine $g(y)$, for $y>4$.

M339J: February 17th, 2020.

(A) $\dfrac{4}{y^2}$

(B) $\dfrac{8}{y^{3/2}}$

(C) $\dfrac{8}{y^3}$

(D) $\dfrac{16}{y}$

(E) $\dfrac{1024}{y^5}$

✱ **72.** An investment account earns an annual interest rate R that follows a uniform distribution on the interval (0.04, 0.08). The value of a 10,000 initial investment in this account after one year is given by $V=10{,}000e^R$.

Let F be the cumulative distribution function of V.

Determine $F(v)$ for values of v that satisfy $0<F(v)<1$.

(A) $\dfrac{10{,}000e^{v/10{,}000}-10{,}408}{425}$

(B) $25e^{v/10{,}000}-0.04$

(C) $\dfrac{v-10{,}408}{10{,}833-10{,}408}$

(D) $\dfrac{25}{v}$

(E) $25\left[\ln\left(\dfrac{v}{10{,}000}\right)-0.04\right]$

1.

$\underline{\underline{\mathbb{P}71}}$: The c.d.f. of Y is, for $y>4$,

$$F_Y(y) = \mathbb{P}[Y \leq y] = \mathbb{P}[T^2 \leq y]$$

$$= \mathbb{P}[T \leq \sqrt{y}] = F_T(\sqrt{y})$$

$$= 1 - \frac{4}{(\sqrt{y})^2} = 1 - \frac{4}{y} \quad , \text{ for } y>4.$$

$$\Rightarrow \quad g(y) = f_Y(y) = F_Y'(y) = -4 \cdot (-1) \cdot y^{-2}$$

$$g(y) = \frac{4}{y^2} \Rightarrow (A)$$

$\boxed{\mathbb{P}72:}$

$$F_Y(v) = \mathbb{P}[Y \leq v]$$

$$= \mathbb{P}[10{,}000e^R \leq v]$$

$$= \mathbb{P}[R \leq \ln(v) - \ln(10{,}000)]$$

$$= \int_{0.04}^{\ln(v)-\ln(10{,}000)} \frac{1}{0.04}\, du =$$

$$= 25\left(\ln(v) - \ln(10{,}000) - 0.04\right)$$

$\Rightarrow$ (E) ∎

182. An insurance company insures red and green cars. An actuary compiles the following data:

Color of car	Red	Green
Number insured	300	700
Probability an accident occurs	0.10	0.05
Probability that the claim exceeds the deductible, given an accident occurs from this group	0.90	0.80

The actuary randomly picks a claim from all claims that exceed the deductible.

Calculate the probability that the claim is on a red car.

(A) 0.300
(B) 0.462
(C) 0.491
(D) 0.667
(E) 0.692

* **183.** X is a random variable with probability density function

$$f(x) = \begin{cases} e^{-2x}, & x \geq 0 \\ 2e^{4x}, & x < 0 \end{cases}$$

Let $T = X^2$.

Determine the probability density function for T for positive values of t.

(A) $f(t) = \dfrac{e^{-2\sqrt{t}}}{2\sqrt{t}} + \dfrac{e^{-4\sqrt{t}}}{\sqrt{t}}$

(B) $f(t) = \dfrac{e^{-2\sqrt{t}}}{2\sqrt{t}}$

(C) $f(t) = e^{-2t} + 2e^{-4t}$

(D) $f(t) = 2te^{-2t^2} + 4te^{-4t^2}$

(E) $f(t) = 2te^{-2t^2}$

4.

→: The c.d.f. of T is

$$F_T(t) = \mathbb{P}[T \leq t] = \mathbb{P}[X^2 \leq t]$$

$$= \mathbb{P}[|X| \leq \sqrt{t}]$$

$$= \mathbb{P}[-\sqrt{t} \leq X \leq \sqrt{t}]$$

$$= \int_{-\sqrt{t}}^{0} f_X(x)\,dx + \int_{0}^{\sqrt{t}} f_X(x)\,dx$$

$$= \int_{-\sqrt{t}}^{0} 2e^{4x}\,dx + \int_{0}^{\sqrt{t}} e^{-2x}\,dx$$

$$= 2 \cdot \frac{1}{4} e^{4x} \Big|_{x=-\sqrt{t}}^{0} + \left(-\frac{1}{2}\right) e^{-2x} \Big|_{x=0}^{\sqrt{t}}$$

$$= \frac{1}{2} - \frac{1}{2} e^{-4\sqrt{t}} + \left(-\frac{1}{2}\right)\left(e^{-2\sqrt{t}} - 1\right)$$

$$= 1 - \frac{1}{2} e^{-4\sqrt{t}} - \frac{1}{2} e^{-2\sqrt{t}}$$

$$\Rightarrow f_T(t) = -\frac{1}{2} e^{-4\sqrt{t}} (-4)\left(\frac{1}{2}\right) \cdot \frac{1}{\sqrt{t}}$$

$$- \frac{1}{2} e^{-2\sqrt{t}} (-2) \cdot \left(\frac{1}{2}\right) \cdot \frac{1}{\sqrt{t}}$$

$$= \frac{e^{-2\sqrt{t}}}{2\sqrt{t}} + \frac{e^{-4\sqrt{t}}}{\sqrt{t}} \Rightarrow (A) \blacksquare$$

Mixing.

Start w/ a random variable Λ which is going to play the role of the mixing parameter. Let it be continuous w/ p.d.f. f_Λ.

Suppose that, conditional on $\Lambda = \lambda$, the random variable X has the p.d.f.

$$f_{X|\Lambda}(x|\lambda)$$

and the c.d.f. $F_{X|\Lambda}(x|\lambda)$.

Then, the unconditional p.d.f. of X is

$$f_X(x) = \int f_{X|\Lambda}(x|\lambda) \cdot f_\Lambda(\lambda)\, d\lambda .$$

Also, the unconditional c.d.f. of X is

$$F_X(x) = \int F_{X|\Lambda}(x|\lambda)\, f_\Lambda(\lambda)\, d\lambda$$

For the moments, we have:

$$\mathbb{E}[X^k] = \mathbb{E}[\mathbb{E}[X^k \mid \Lambda]] .$$

The variance equals:

$$\mathrm{Var}[X] = \mathbb{E}[\mathrm{Var}[X|\Lambda]] + \mathrm{Var}[\mathbb{E}[X|\Lambda]] .$$