207. For an insurance:

M339J: February 24th, 2020.

(i) Losses have density function

$$f(x) = \begin{cases} 0.02x, & 0 < x < 10 \\ 0, & \text{elsewhere} \end{cases}$$

(ii) The insurance has an ordinary deductible of 4 per loss. $d = 4$

(iii) Y^P is the claim payment per payment random variable.

Calculate $E[Y^P]$.

(A) 2.9

(B) 3.0

(C) 3.2

(D) 3.3

(E) 3.4

$$\mathbb{E}[Y^P] \underset{\text{by def'n}}{=} \int_0^6 y \cdot f_{Y^P}(y)\,dy$$

corresponds to the amt of the loss

$$f_{Y^P}(y) = \frac{f_X(y+4)}{S_X(4)}$$

corresponds to pmt by insurer given it happens

208. DELETED

$$F_X(4) = \int_0^4 0.02x\,dx = 0.02 \cdot \frac{x^2}{2}\Big|_{x=0}^{4}$$

$$= 0.16 \quad \Rightarrow \quad S_X(4) = 0.84$$

$$f_{Y^P}(y) = \frac{0.02(y+4)}{0.84} = \cancel{0.024(y+4)} = 0.0238(y+4)$$

$$\Rightarrow \mathbb{E}[Y^P] = \int_0^6 y\,(0.0238)(y+4)\,dy =$$

$$= 0.0238 \int_0^6 (y^2 + 4y)\,dy$$

$$= 0.0238 \left[\frac{y^3}{3} + 4 \cdot \frac{y^2}{2} \right]_{y=0}^{6} = 3.427 \Rightarrow (E)$$

1.

*** 168.** For an insurance:

(i) Losses can be 100, 200 or 300 with respective probabilities 0.2, 0.2, and 0.6.

(ii) The insurance has an ordinary deductible of 150 per loss.

(iii) Y^P is the claim payment per payment random variable.

Calculate $\mathrm{Var}(Y^P)$.

(A) 1500

(B) 1875

(C) 2250

(D) 2625

(E) 3000

169. The distribution of a loss, X, is a two-point mixture:

(i) With probability 0.8, X has a two-parameter Pareto distribution with $\alpha = 2$ and $\theta = 100$.

(ii) With probability 0.2, X has a two-parameter Pareto distribution with with $\alpha = 4$ and $\theta = 3000$.

Calculate $\Pr(X \leq 200)$.

(A) 0.76

(B) 0.79

(C) 0.82

(D) 0.85

(E) 0.88

2.

→ :

$$X \sim \begin{cases} 100 & \text{w/ probab. } 0.2 \\ 200 & \text{w/ probab. } 0.2 \\ 300 & \text{w/ probab. } 0.6 \end{cases}$$

⇒ the <u>per-loss</u> random variable:

$$Y^L = (X-150)_+ \sim \begin{cases} 0 & \text{w/ probab. } 0.2 \\ 50 & \text{w/ probab. } 0.2 \\ 150 & \text{w/ probab. } 0.6 \end{cases}$$

⇒ the <u>per-payment</u> random variable

$$Y^P = X-150 \mid X>150 \sim \begin{cases} 50 & \text{w/ probab. } 0.25 \\ 150 & \text{w/ probab. } 0.75 \end{cases}$$

$$\mathbb{E}[Y^P] = 0.25(50) + 0.75(150) = 125$$

$$\mathbb{E}[(Y^P)^2] = 0.25(50)^2 + 0.75(150)^2 = 17,500$$

$$\Rightarrow \operatorname{Var}[Y^P] = 17,500 - (125)^2 = 1,875 \Rightarrow \text{(B)}$$ ■

Example.

$X \sim$ Exponential (mean $= \theta$)

$d > 0$ is the ordinary deductible

Per-loss: $Y^L = (X-d)_+ = \max(X-d, 0)$

Per-payment: $Y^P = X-d \mid X>d$

Q: What is the dist'n of Y^P?

→: Any $y > 0$:

$$S_{Y^P}(y) = \mathbb{P}[Y^P > y]$$

$$= \mathbb{P}[X-d > y \mid X > d]$$

$$= \frac{\mathbb{P}[X > d+y, \cancel{X > d}]}{\mathbb{P}[X>d]}$$

$$= \frac{\mathbb{P}[X > d+y]}{\mathbb{P}[X>d]} = \frac{S_X(d+y)}{S_X(d)}$$

$$\Rightarrow S_{Y^P}(y) = \frac{e^{-\frac{\cancel{d}+y}{\theta}}}{e^{-\frac{\cancel{d}}{\theta}}} = e^{-\frac{y}{\theta}}$$

$\Rightarrow$

$Y^P \sim$ Exponential (mean $= \theta$)

The MEMORYLESS PROPERTY of the exponential dist'n.

21. A claim severity distribution is exponential with mean 1000. An insurance company will pay the amount of each claim in excess of a deductible of 100.

Calculate the variance of the amount paid by the insurance company for one claim, including the possibility that the amount paid is 0.

$Var[Y^L] = ?$

(A) 810,000

(B) 860,000

(C) 900,000

(D) 990,000

(E) 1,000,000

$$Y^L = \begin{cases} 0 & X \leq d \quad \text{w/ probab. } F_X(d) \\ X-d & X>d \quad \text{w/ probab. } S_X(d) \end{cases}$$

$$\mathbb{E}[Y^L] = 0 \cdot F_X(d) + \underbrace{\mathbb{E}[X-d \mid X>d]}_{= \theta} \cdot S_X(d)$$

$$\cdot \ \mathbb{E}[Y^L] = \underbrace{1000}_{\theta} \underbrace{e^{-\frac{100}{1000}}}_{S_X(100)} = 1000\, e^{-0.1}$$

$$\cdot \ \mathbb{E}[(Y^L)^2] = \underbrace{\mathbb{E}[(X-d)^2 \mid X>d]}_{2\theta^2} \cdot S_X(d)$$

$$= 2\theta^2 \cdot S_X(d)$$

$$\Rightarrow \mathbb{E}[(Y^L)^2] = 2 \cdot 10^6 \cdot e^{-0.1}$$

$$\Rightarrow Var[Y^L] = 2 \cdot 10^6 \cdot e^{-0.1} - 10^6 e^{-0.2}$$

$$= 990{,}944 \Rightarrow (D) \ \blacksquare$$

Problem. Let the ground-up loss X be modeled by a two-parameter Pareto w/ parameters $\alpha = 3$ and $\theta = 200$.
There is a FRANCHISE DEDUCTIBLE of 200.
Find $\mathbb{E}[Y^P]$.

→:

$$\mathbb{E}[Y^P] = d + \frac{\mathbb{E}[X] - \mathbb{E}[X \wedge d]}{S_X(d)}$$

w/ $X \sim \text{Pareto}(\alpha = 3, \theta = 200)$.

- $S_X(d) = \left(\frac{\theta}{d+\theta}\right)^{\alpha}$

- $\mathbb{E}[X] = \frac{\theta^1 \cdot 1!}{\alpha - 1} = \frac{\theta}{\alpha - 1}$

- $\mathbb{E}[X \wedge d] = \frac{\theta}{\alpha - 1}\left(1 - \left(\frac{\theta}{d+\theta}\right)^{\alpha - 1}\right)$

$$\Rightarrow \mathbb{E}[Y^P] = d + \frac{\cancel{\frac{\theta}{\alpha-1}} - \cancel{\frac{\theta}{\alpha-1}}\left(\cancel{1} - \left(\frac{\theta}{d+\theta}\right)^{\cancel{\alpha-1}\,1}\right)}{\left(\frac{\theta}{d+\theta}\right)^{\cancel{\alpha}\,1}}$$

$$= d + \frac{\frac{\theta}{\alpha-1}}{\frac{\theta}{d+\theta}} = d + \frac{d+\theta}{\alpha - 1}$$

$$\Rightarrow \mathbb{E}[Y^P] = 200 + \frac{200 + 200}{3 - 1} = 400 \blacksquare$$