100. The unlimited severity distribution for claim amounts under an auto liability insurance policy is given by the cumulative distribution:

$$F(x) = 1 - 0.8e^{-0.02x} - 0.2e^{-0.001x}, \quad x \geq 0$$

The insurance policy pays amounts up to a limit of 1000 per claim.

Calculate the expected payment under this policy for one claim.

(A) 57

(B) 108

(C) 166

(D) 205

(E) 240

M339J: February 5th, 2020.

* **101.** The random variable for a loss, X, has the following characteristics:

x	$F(x)$	$E(X \wedge x)$
0	0.0	0
100	0.2	91
200	0.6	153
1000	1.0	331

Calculate the mean excess loss for a deductible of 100.

$d = 100$

(A) 250

(B) 300

(C) 350

(D) 400

(E) 450

1.

$$\mathbb{E}[X-d \mid X>d] = \frac{\mathbb{E}[(X-d)\cdot\mathbb{I}_{[X>d]}]}{\mathbb{P}[X>d]} = \frac{\mathbb{E}[(X-d)_+]}{S_X(d)} = \frac{\mathbb{E}[(X-d)_+]}{1-F_X(d)}$$

In general:

$$(X-d)_+ = X - X \wedge d$$

In this problem:

$$\text{answer} = \frac{\mathbb{E}[X] - \mathbb{E}[X \wedge 100]}{1 - F_X(100)} = \frac{331 - 91}{1 - 0.2}$$

$$= 300 \Rightarrow (B) \blacksquare$$

Note that $F_X(1000) = 1$, i.e.,

$$\mathbb{P}[X \leq 1000] = 1$$

$$\Rightarrow \underbrace{X \wedge 1000}_{\substack{\shortparallel \\ \begin{cases} X & \text{if } X \leq 1000 \\ 1000 & \text{if } X > 1000 \end{cases}}} = X$$

an impossible event

$\blacksquare$

$X \sim \text{Exponential}(\theta = 10^4)$

13. The loss severity random variable X follows the exponential distribution with mean 10,000.

Determine the coefficient of variation of the excess loss variable $Y^L = \max(X - 30000, 0)$.

by def'n

$$\frac{SD[Y^L]}{\mathbb{E}[Y^L]}$$

$Y^L = (X - 3\cdot 10^4)_+$

(A) 1.0

(B) 3.0

(C) 6.3

(D) 9.0

(E) 39.2

$$\mathbb{E}[Y^L] = \mathbb{E}[(X-3\cdot 10^4)_+] =$$

$$= \mathbb{E}[(X-3\cdot 10^4)\cdot \mathbb{I}_{[X>3\cdot 10^4]}]$$

$$= \int_{30K}^{+\infty} \underbrace{(x-30K)}_{} \frac{1}{10K} e^{-\frac{x}{10K}} dx$$

$u = x - 30K$ $\begin{cases} x = u + 30K \\ dx = du \end{cases}$

$$= \int_0^{+\infty} u \cdot \frac{1}{10K} e^{-\frac{u+30K}{10K}} du$$

$$= e^{-3} \boxed{\int_0^{+\infty} u \cdot \frac{1}{10K} e^{-\frac{u}{10K}} du} = 10^4 e^{-3}$$

$= 10K$

3.

Alternatively :

$$\mathbb{E}[Y^L] = \mathbb{E}[(X-3\cdot 10^4)_+]$$
$$= \mathbb{E}[X] - \mathbb{E}[X \wedge 3\cdot 10^4]$$
$$= \theta - \theta(1-e^{-3}) = \theta e^{-3} = 10^4 e^{-3}$$

↑ Using STAM Tables

$$\mathbb{E}[(Y^L)^2] = \int_{30K}^{+\infty} \underbrace{(x-30K)}^{2} \cdot \frac{1}{10K} e^{-\frac{x}{10K}} \, dx$$

$\to u = x - 30K$ $\begin{cases} x = u + 30K \\ dx = du \end{cases}$

$$= \int_0^{+\infty} u^2 \cdot \frac{1}{10K} e^{-\frac{u+30K}{10K}} \, du$$

density of exponential dist'n

$$= e^{-3} \underbrace{\int_0^{+\infty} u^2 \cdot \frac{1}{10K} e^{-\frac{u}{10K}} \, du}_{\text{the 2nd moment of the exponential with } \theta = 10K} = e^{-3} \cdot 2 \cdot 10^8$$

$$\Rightarrow \mathrm{Var}[Y^L] = 2\cdot 10^8 \cdot e^{-3} - (10^4 e^{-3})^2$$
$$= 2e^{-3} \cdot 10^8 - 10^8 e^{-6}$$

4.

$$\Rightarrow \text{coeff. of. variation} : \frac{\sqrt{2e^{-3} \cdot 10^8 - 10^8 \cdot e^{-6}}}{10^4 \cdot e^{-3}} \approx 6.259$$

⇓

(C) ∎

160. You are given a random sample of observations:

0.1 0.2 0.5 0.7 1.3

You test the hypothesis that the probability density function is:

$$f(x)=\frac{4}{(1+x)^5}, \quad x>0$$

Calculate the Kolmogorov-Smirnov test statistic.

(A) Less than 0.05

(B) At least 0.05, but less than 0.15

(C) At least 0.15, but less than 0.25

(D) At least 0.25, but less than 0.35

(E) At least 0.35

161. DELETED

* **162.** A loss, X, follows a 2-parameter Pareto distribution with $\alpha=2$ and unspecified parameter θ. You are given:

* $$E[X-100 \mid X>100]=\frac{5}{3}E[X-50 \mid X>50]$$

Calculate $E[X-150 \mid X>150]$.

(A) 150

(B) 175

(C) 200

(D) 225

(E) 250

5.

→: In general, for $X \sim \text{Pareto}(\alpha, \theta)$ and for a deductible d:

$$\mathbb{E}[X-d \mid X>d] = \frac{\mathbb{E}[(X-d)_+]}{S_X(d)}$$

$$= \frac{\mathbb{E}[X] - \mathbb{E}[X \wedge d]}{1-F_X(d)} = \text{The STAM Tables for Pareto}$$

$$= \frac{\cancel{\frac{\theta}{\alpha-1}} - \frac{\theta}{\alpha-1}\left(\cancel{1} - \cancel{\left(\frac{\theta}{d+\theta}\right)^{\alpha-1}}\right)}{\left(\frac{\theta}{d+\theta}\right)^{\cancel{\alpha}}}$$

$$= \frac{\frac{\theta}{\alpha-1}}{\frac{\theta}{d+\theta}} = \frac{d+\theta}{\alpha-1} = d+\theta$$

In this problem! $\alpha=2$

(*)

$$100+\theta = \frac{5}{3}(50+\theta) \Rightarrow$$

$$\Rightarrow 300 + 3\theta = 250 + 5\theta \Rightarrow \theta = 25$$

⇒ Answer: $150+25 = 175 \Rightarrow$ (B)