M339J: February 7^{th}, 2020.

Problem. Find the ratio of the 90^{th} percentile to the median of the dist'n w/ the density f'tion of the form

$$f_X(x) = \kappa e^{-0.1x}, \quad x > 0.$$

→ : Observe $X \sim$ Exponential (mean $= \theta = 10$).

$$\Rightarrow \quad F_X(x) = 1 - e^{-0.1x}$$

Integrate or use the STAM Table

By def'n: The $100p^{th}$ percentile is any π_p such that

$$F_X(\pi_p -) \leq p \leq F_X(\pi_p).$$

If we have $f_X(x) > 0$ for every $x > 0$, then F_X is strictly increasing

$\Rightarrow$ F_X has an inverse

$$\Rightarrow \quad \pi_p = F_X^{-1}(p)$$

✱ **80.** A charity receives 2025 contributions. Contributions are assumed to be mutually independent and identically distributed with mean 3125 and standard deviation 250.

Calculate the approximate 90^{th} percentile for the distribution of the total contributions received.

(A) 6,328,000
(B) 6,338,000
(C) 6,343,000
(D) 6,784,000
(E) 6,977,000

81. Claims filed under auto insurance policies follow a normal distribution with mean 19,400 and standard deviation 5,000.

Calculate the probability that the average of 25 randomly selected claims exceeds 20,000.

(A) 0.01
(B) 0.15
(C) 0.27
(D) 0.33
(E) 0.45

82. An insurance company issues 1250 vision care insurance policies. The number of claims filed by a policyholder under a vision care insurance policy during one year is a Poisson random variable with mean 2. Assume the numbers of claims filed by different policyholders are mutually independent.

Calculate the approximate probability that there is a total of between 2450 and 2600 claims during a one-year period?

(A) 0.68
(B) 0.82
(C) 0.87
(D) 0.95
(E) 1.00

3.

$\rightarrow$: $S = X_1 + X_2 + \cdots + X_{2025}$

By the CLT,

"$$\frac{S - \mathbb{E}[S]}{SD[S]} \approx N(0,1)$$"

- $\mathbb{E}[S] = 2025(3125) = 6{,}328{,}125$
- $Var[S] = 2025 \cdot Var[X]$

$\Rightarrow SD[S] = 45 \cdot SD[X] = 45(250) = 11{,}250$

<u>Note</u>: The percentile of a normal dist'n is the linear transform of the (same) percentile of the std normal dist'n.

In this problem, the 90^{th} percentile of the std normal is 1.28

$\Rightarrow$ the 90^{th} percent of S is approximately

$$6{,}328{,}125 + 11{,}250(1.28) = 6{,}342{,}525$$

$\Rightarrow$ (C) ∎

Focus on the p.g.f.

Consider $\mathbb{N}_0$-valued random variables, i.e., look @ rnd variables w/ support contained within $\mathbb{N}_0 = \{0, 1, 2, \ldots\}$.

Denote such a random variable by N.

By def'n:

$$P_N(z) = \mathbb{E}[z^N]$$

$$= \sum_{n=0}^{+\infty} z^n \cdot p_N(n)$$

$$= z^0 \cdot p_N(0) + z^1 \cdot p_N(1) + \cdots + z^n \cdot p_N(n) + \cdots$$

$$\Rightarrow \begin{cases} P_N(0) = p_N(0) \\ P_N(1) = 1 \\ P_N'(0) = p_N(1) \end{cases}$$