UNIVERSITY OF TEXAS AT AUSTIN

HW Assignment 1

Note: You **must** show all your work. Numerical answers without a proper explanation or a clearly written down path to the solution will be assigned zero points.

Problem 1.1. (5 points) Let $\Omega = \{a_1, a_2, a_3, a_4, a_5\}$ be an outcome space, and let $\mathbb{P}$ be a probability distribution on Ω. Assume that $\mathbb{P}[A] = 0.5$, $\mathbb{P}[B] = 0.4$, $\mathbb{P}[C] = 0.4$, and $\mathbb{P}[D] = 0.2$, where

$$A = \{a_1, a_2, a_3\},\ B = \{a_2, a_3, a_4\},$$
$$C = \{a_3, a_5\} \text{ and } D = \{a_4\}.$$

Are the events A and B independent?

Problem 1.2. (10 points) Consider a set-up in which a transmitter is transmitting either a 0 or a 1 and the receiver indicates that it received either a 0 or a 1. Denote the events that $i = 0, 1$ was transmitted by T_i, and the events that $i = 0, 1$ was indicated as received by R_i.
It is possible to have transmission errors. In fact, you are given the following data on accuracy and the frequency of transmitted signals:

$$\mathbb{P}[R_0 \,|\, T_0] = 0.99,\ \mathbb{P}[R_1 \,|\, T_1] = 0.98,$$

and

$$\mathbb{P}[T_0] = 0.75.$$

(a) Given that the receiver indicated 1, what is the probability that there was an error in the transmission?
(b) What is the overall probability that there was an error in transmission?

Problem 1.3. (10 points) Two people are picked at random from a group of 50 and given \$10 each. After that, independently of what happened before, three people are picked from the same group - one or more people could have been picked both times - and given \$10 each. What is the probability that at least one person received \$20?

Problem 1.4. (5 points) Write down the definition of the *cumulative distribution function* of a random variable.

Problem 1.5. (10 points) Two coins are tossed and a (6-sided) die is rolled. Describe a sample space (probability space), together with the probability, on which such a situation can be modelled. Find the probability mass function of the random variable whose value is the sum of the number on the die and the total number of heads.

Problem 1.6. (10 points) A continuous random variable X has the probability density function f_X given by

$$f_X(x) = A - \tfrac{x}{50}, \quad 0 \le x \le 10.$$

(a) Find the value of the constant A.
(b) Find the value of the survival function of X at 7, i.e., calculate $S_X(7)$.