M339J Spring 2020
University of Texas at Austin
HW #2
Instructor: Milica Čudina

Note: You **must** show all your work for the free-response problems. Numerical answers without a proper explanation or a clearly written down path to the solution will be assigned zero points.
For the true/false questions and multiple-choice problems, no partial credit will be given.

2.1. **True/false questions.**

Problem 2.1. (2 points) For a random variable X and for a positive constant d, in our usual notation, we have

$$\mathbb{E}[X] = e_X(d)S_X(d) + \mathbb{E}[X \wedge d]\,. \tag{2.1}$$

True or false?

Problem 2.2. (2 pts) Let X have a Pareto distribution with parameters α and θ. Then, $\mathbb{E}[X^k]$ exists for every $k = 1, 2, \dots$. *True or false?*

Problem 2.3. (2 pts) Let the random variable X represent the outcome of rolling a fair dodecahedron (i.e., a 12 sided "die") with faces numbered 1 through 12. Then the expectation of X equals 6. *True or false?*

2.2. **Free-response problems.**

Problem 2.4. (5 points) A nonnegative random variable X has a hazard rate function of

$$h_X(x) = A + e^{2x}, \qquad x \geq 0.$$

You are given that $S_X(0.4) = 0.5$.
Determine the value of A.

Problem 2.5. (5 points) *Source: Problem 3.6 from the textbook.*
For two random variables, X and Y, you are given that

$$e_Y(30) = e_X(30) + 4.$$

Let X have a uniform distribution on the interval from 0 to 100, and let Y have a uniform distribution on the interval from 0 to w.
Determine w.

Problem 2.6. (5 points) Let $X \sim Pareto(\alpha = 3, \theta = 3000)$. Assume that there is a deductible of $d = 5000$. Find $\mathbb{E}[X \wedge d]$.

Problem 2.7. (10 points) Let X be the ground-up loss random variable whose density is given by

$$f_X(x) = \begin{cases} 0.01, & 0 \leq x \leq 80, \\ 0.03 - 0.00025x, & 80 < x \leq 120. \end{cases}$$

Let there be an ordinary deductible of $d = 20$.
Calculate $\mathbb{E}[X \wedge d]$.

2.3. **Multiple-choice questions.**

Problem 2.8. (5 pts) Let the severity random variable X be continuous such that $f_X(x) > 0$ for all $x > 0$. Let Y^P denote the per payment random variable associated with X for some ordinary deductible d. Then the random variable Y^P is

(a) continuous.
(b) discrete, but not degenerate (constant).
(c) degenerate (constant).
(d) mixed.
(e) None of the above

Problem 2.9. (5 pts) The ground-up loss X is modeled by an exponential distribution with mean \$500. There is an ordinary deductible of $d = 200$. What can you say about the expected value of the **per-loss** random variable?

(a) It is less than 100.
(b) It is more than 100, but less than 250.
(c) It is more than 250, but less than 375.
(d) It is more than 375, but less than 500.
(e) None of the above

Problem 2.10. (5 pts) Given that the hazard rate function of a random variable X equals $h_X(x) = 0.3x$ for $x > 0$, and defining the density function f_X as we did in class, we can conclude that

(a) $0 \leq f_X(2) < 1/4$
(b) $1/4 \leq f_X(2) < 1/3$
(c) $1/3 \leq f_X(2) < 1/2$
(d) $1/2 \leq f_X(2) < 1$
(e) None of the above.

Problem 2.11. (5 pts) Given that the hazard rate function of a random variable X equals $h_X(x) = 1/x$ for $x > 1$, we can conclude that

(a) $0 \leq f_X(2) < 1/4$
(b) $1/4 \leq f_X(2) < 1/3$
(c) $1/3 \leq f_X(2) < 1/2$
(d) $1/2 \leq f_X(2) < 1$
(e) None of the above.