M339J *Probability models*
University of Texas at Austin
HW III
Instructor: Milica Čudina

Note: You **must** show all your work. Numerical answers without a proper explanation or a clearly written down path to the solution will be assigned zero points.

Problem 3.1. (10 points) *Source: Problem 4.3 from the textbook.*
Assume that the claims r.v. X has a Pareto distribution with $\alpha = 2$ and θ unknown.
Claims for the following year are denoted by Y and will experience uniform inflation of 6%.

(i) (2 points) Find the expression for the probability $\mathbb{P}[X > d]$ in terms of d and θ.
(ii) (3 points) Find the expression for the probability $\mathbb{P}[Y > d]$ in terms of d and θ.
(iii) (5 points) Find the expression for the ratio $\rho(d) = \frac{\mathbb{P}[X>d]}{\mathbb{P}[Y>d]}$ in terms of d and θ.
Find the limit of $\rho(d)$ as $d \to \infty$.

Problem 3.2. (10 points) Let X have a two-point mixture distribution. More precisely, with probability 1/3, X has the Pareto distribution with parameters $\alpha = 3$ and $\theta = 10$ and with probability 2/3, X has the Gamma distribution with parameters $\alpha = 2$ and $\theta = 8$.

Find $Var[X]$.

Problem 3.3. (10 points) *Source: Sample C Exam Problem #100.*
Let X have the following cumulative distribution function

$$F_X(x) = 1 - 0.8e^{-0.02x} - 0.2e^{-0.001x}, \qquad x \geq 0.$$

Let $u = 1000$.
Find $\mathbb{E}[X \wedge u]$.

Problem 3.4. (10 points) Let Y be lognormal with parameters $\mu = 1$ and $\sigma = 2$.
Define $\tilde{Y} = 3Y$.
Find the median of $\tilde{Y}$, i.e., find the value m such that $\mathbb{P}[\tilde{Y} \leq m_Y] = 1/2$.

Problem 3.5. (10 points) In the notation of our tables, let X be a Weibull random variable with parameters $\theta = 20$ and $\tau = 2$.
Define $Y = 5X$ and denote the coefficient of variation of Y by CV_Y. Find CV_Y.
Hint: The following facts you may have forgotten from probability could be useful:

$$\Gamma(1/2) = \sqrt{\pi},$$
$$\Gamma(1) = 1,$$
$$\Gamma(\alpha + 1) = \alpha\Gamma(\alpha), \quad \text{for all } \alpha.$$