Due date Friday, October 18^{th}
M339J: Spring 2020
University of Texas at Austin
HW IV
Instructor: Milica Čudina

Note: You **must** show all your work. Numerical answers without a proper explanation or a clearly written down path to the solution will be assigned zero points.

Problem 4.1. (5 points) Provide the definition of a *continuous random variable.*

Problem 4.2. (5 points) Provide an example of a continuous random variable. Justify your choice.

Problem 4.3. (5 points) Provide the definition of a *discrete random variable.*

Problem 4.4. (5 points) Provide an example of a discrete random variable. Justify your choice.

Problem 4.5. (15 points) Total claims for a health plan have a Pareto distribution with $\alpha = 2$, and $\theta = 500$.

The health plan implements an incentive to physicians that will pay for every claim a bonus of 50% of the amount by which that claim is less than 500; otherwise no bonus is paid.

It is anticipated that with the incentive plan, the claim distribution will become Pareto with $\alpha = 2$ and $\theta = K$.

With the new distribution, it turns out that the expected claims plus the expected bonus equals the expected claims prior to the bonus system.

Determine the value of K.

Problem 4.6. (8 points) In year a, the expected number of claims is $5,000$. Individual losses in a year are modeled by a r.v. X which is assumed to be Pareto distributed with $\alpha = 2$ and $\theta = 2,000$.

(i) (3 points) A reinsurer pays the excess of each individual loss over 3,000. What is the reinsurer's expected cost per loss?

(ii) (2 points) For the above the reinsurer is paid a premium equal to 110% of expected number of losses $\times$ expected cost per loss. What is the reinsurer's premium?

(iii) (3 points) In year b, losses will experience 5% inflation, but the frequency of losses will not change. The reinsurer's premium is calculated in the same fashion as in year a. What is the ratio of the reinsurer's premium in year b to the reinsurer's premium in year a?

Problem 4.7. (2 pts) The ground-up loss random variable is denoted by X. An insurance policy on this loss has a franchise deductible of d and no policy limit. Then, the expected **policyholder** payment per loss equals

$$\mathbb{E}[X \wedge d].$$

True or false?

Problem 4.8. (5 pts) Let the loss random variable X be Pareto with $\alpha = 3$ and $\theta = 5000$. There is a franchise deductible of $d = 1000$.

Then, in our usual notation,

(a) $3,500 \leq \mathbb{E}[Y^P] < 4,500$
(b) $4,500 \leq \mathbb{E}[Y^P] < 5,500$
(c) $5,500 \leq \mathbb{E}[Y^P] < 6,500$
(d) $6,500 \leq \mathbb{E}[Y^P] < 7,500$
(e) None of the above