HOMEWORK ASSIGNMENT #5

Note: You **must** show all your work. Numerical answers without a proper explanation or a clearly written down path to the solution will be assigned zero points.

Problem 5.1. (6 points) Losses have an exponential distribution with a mean of 1,000. There is a deductible of 500. Determine the amount by which the deductible should be raised in order to double the loss elimination ratio.

Problem 5.2. (10 points) *Source: Sample C Exam Problem #87.*
Let X be the ground-up loss random variable whose density is given by

$$f_X(x) = \begin{cases} 0.01, & 0 \leq x \leq 80, \\ 0.03 - 0.00025x, & 80 < x \leq 120. \end{cases}$$

Let there be an ordinary deductible of $d = 20$.
Calculate the **loss elimination ratio.**

Problem 5.3. (10 points) Let the ground-up loss X be exponentially distributed with mean \$500.
An insurance policy has an ordinary deductible of \$50 and a policy limit of \$2000.
Find the expected value of the amount paid (by the insurance company) per positive payment.

Problem 5.4. (2 pts) The Poisson distribution has the memoryless property. *True or false?*

Problem 5.5. (2 pts) Let S_1 and S_2 be independent and have Poisson distribution with parameters λ_1 and λ_2, respectively. Then, $S = S_1 + S_2$ has a Poisson distribution with the parameter $\lambda = \lambda_1 + \lambda_2$. *True or false?*

Problem 5.6. (5 points) Let the number of floods in a calendar year be denoted by N and modeled using the Poisson distribution with mean 5. We say that a flood is "minor" if the damages associated with it do not exceed \$1,000,000. Otherwise, a flood is designated as "major". The number of floods and the damages caused by individual floods are assumed to be independent.

Assume that the probability that an observed flood is "major" equals 1/5.

Find the probability that the number of "major" floods is 2, given that the **total** number of floods in that year equals 5.

Problem 5.7. (5 points) *Source: Prof. Jim Daniel, personal communication.*
Let the number of car accidents in a calendar year by a group of drivers be denoted by N and modeled using the Poisson distribution with mean 10.
Assume that the probability that the damage in any single accident is at most \$1,000 equals 2/5.
The number of accidents and the damages caused are assumed to be independent.
Find the probability that the number of accidents in one year with damage greater than \$1000 is 5, given that the number of accidents in that year with damage at most \$1000 equals 100.

Problem 5.8. (5 pts) Let the random variable N be in the $(a, b, 0)$ class with $a = 0$ and $b = 8$. Find $\mathbb{P}[N = 10]$.

Problem 5.9. (5 pts) Let N^T be a zero-truncated Poisson random variable with parameter $\lambda = 4$. What is $Var[N^T]$?