

*Note:* You **must** show all your work. Numerical answers without a proper explanation or a clearly written down path to the solution will be assigned zero points.

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**Problem 7.1.** (10 points) Aggregate losses  $S$  under an insurance policy follow a compound Poisson process with mean equal to 1, and a severity random variable  $X$ . The support of the random variable  $X$  is  $\{10, 20\}$ . Moreover, we are given that

$$\mathbb{P}[X = 10] = 3\mathbb{P}[X = 20].$$

The premium for this policy equals 22.

If the insurance company makes a profit, i.e., if the premium exceeds the aggregate losses, it pays a dividend to the policyholder equal to one-third of the profit (the excess of the premium over the aggregate losses). Find the expected dividend.

**Problem 7.2.** (10 points) *Source: Based on Problem #165 from sample STAM Exam.*

Consider the following collective risk model:

- (i) The claim count random variable  $N$  is Poisson with mean 3.
- (ii) The severity random variable has the following probability (mass) function:

$$p_X(1) = 0.6, p_X(2) = 0.4.$$

- (iii) As usual, individual loss random variables are mutually independent and independent of  $N$ .

Assume that an insurance covers **aggregate losses** subject to a deductible  $d = 3$ .

Find the expected value of aggregate payments for this insurance.

**Problem 7.3.** (10 pts) We are using the aggregate loss model and our usual notation. The frequency random variable  $N$  is assumed to be Poisson distributed with mean equal to 1. The severity random variable is assumed to have the following probability mass function:

$$p_X(100) = 3/5, \quad p_X(200) = 3/10, \quad p_X(300) = 1/10.$$

Find the probability that the total aggregate loss **exactly** equals 300.

**Problem 7.4.** (10 pts) In the compound model for aggregate claims, let the frequency random variable  $N$  have the geometric distribution with mean 4.

Moreover, let the individual losses have the distribution

$$p_X(0) = 1/2, p_X(100) = 1/2.$$

Define the aggregate loss as  $S = \sum_{j=1}^N X_j$ . How much is  $\mathbb{E}[(S - 100)_+]$ ?

**Problem 7.5.** (10 points) In the compound model for aggregate claims, let the frequency random variable  $N$  be negative binomial with parameters  $r = 15$  and  $\beta = 5$ .

Moreover, let the common distribution of the i.i.d. severity random variables  $\{X_j; j = 1, 2, \dots\}$  be the two-parameter Pareto with  $\alpha = 3$  and  $\theta = 10$ .

Let our usual assumptions hold, i.e., let  $N$  be independent of  $\{X_j; j = 1, 2, \dots\}$ . The insurer is interested in finding the total premium  $\pi$  such that the aggregate losses exceed it with the probability less than or equal to 5%. Using the normal approximation, find  $\pi$  such that

$$\mathbb{P}[S > \pi] = 0.05.$$