

*Note:* You **must** show all your work. Numerical answers without a proper explanation or a clearly written down path to the solution will be assigned zero points.

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**Problem 8.1.** (5 points) You want to fit to the observed values

$$4, 5, 7$$

a two-parameter Pareto distribution with parameters  $\alpha = 4$  and  $\theta$  unknown using maximum likelihood estimation. Write down **clearly** an **explicit** expression for the loglikelihood function (of course, as a function of  $\theta$ ).

**Problem 8.2.** (10 points) Consider a random variable  $Y$  such that  $Y = e^X$  with  $X \sim \text{Gamma}(\alpha = 2, \theta)$ . Your colleague was playing with the collected data and the only things you still know about the observations from  $Y$  are:

- (i) There was a total of 20 observations;
- (ii) The product of all observations was 5,000.

Find  $\hat{\theta}_{MLE}$ , i.e., the Maximum Likelihood Estimate of  $\theta$  based on the observed values.

**Problem 8.3.** (5 points) Let  $X_1, X_2, \dots, X_n$  be a random sample from an exponential distribution with an unknown mean  $\theta$ . Consider these as individual, unmodified data. What is the expression for the maximum likelihood estimator of  $\theta$  denoted by  $\hat{\theta}_{MLE}$ ?

**Problem 8.4.** (10 points) *Source: Sample STAM problem #179.* The time to an accident follows an exponential distribution. A random sample of size two has a mean time of 4. Let  $Y$  denote the average of a new sample of size two from the same distribution. Calculate the maximum likelihood estimate of  $\mathbb{P}[Y > 8]$ .

*Hint: Remember that the sum of independent, identically distributed exponential random variables has a gamma distribution. You can convince yourselves of this fact using moment generating functions.*

**Problem 8.5.** (5 points) Assume that the ground-up loss random variable  $X$  is exponential with parameter  $\theta$ .

An insurer imposes a **franchise** deductible of  $d = 100$ . The observed values of the payments are

$$500, 800, 1000.$$

There is no policy limit. What is the maximum likelihood estimate of  $\theta$ ?

**Problem 8.6.** (5 points) Let  $N \sim \text{Poisson}(\lambda)$  for an unknown parameter  $\lambda$ . The following values are observed

$$2, 6, 9, 10.$$

Find the maximum likelihood estimate of  $\mathbb{P}[N \leq 1]$ .

**Problem 8.7.** (10 points) Claim amounts in a particular county are modeled using the Weibull distribution with parameter  $\tau = 2$  and parameter  $\theta$  unknown. A sample of four claim amounts is

$$100, \quad 240, \quad 300, \quad 520$$

There are two additional claims both equal to 800 due to the fact that the policy limit of 800 was exceeded. Calculate the maximum likelihood estimate of the parameter  $\theta$ .