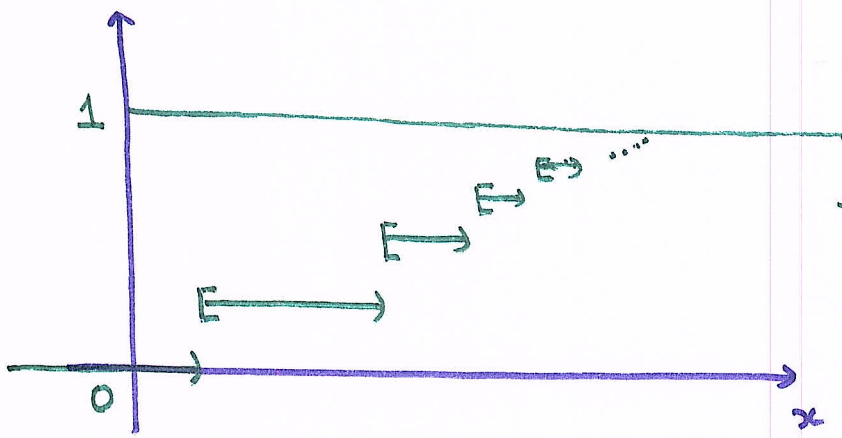
Review:

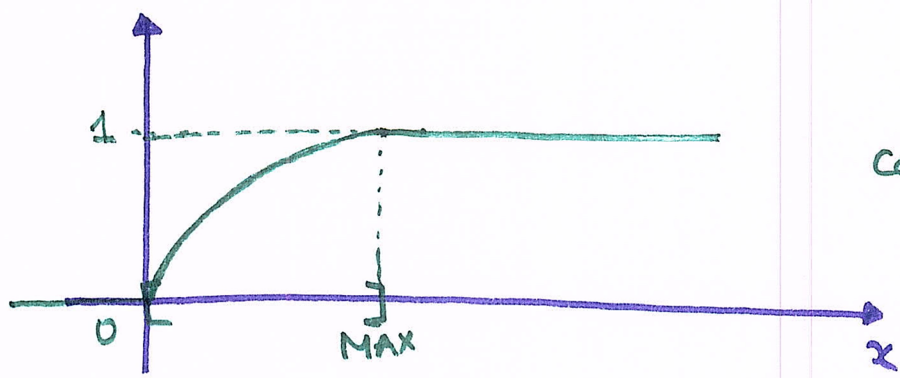
Def'n. For a random variable X, the cumulative dist'n f'tion is defined as $F_X : \mathbb{R} \rightarrow [0,1]$ given by

$$F_X(x) = \mathbb{P}[X \leq x].$$

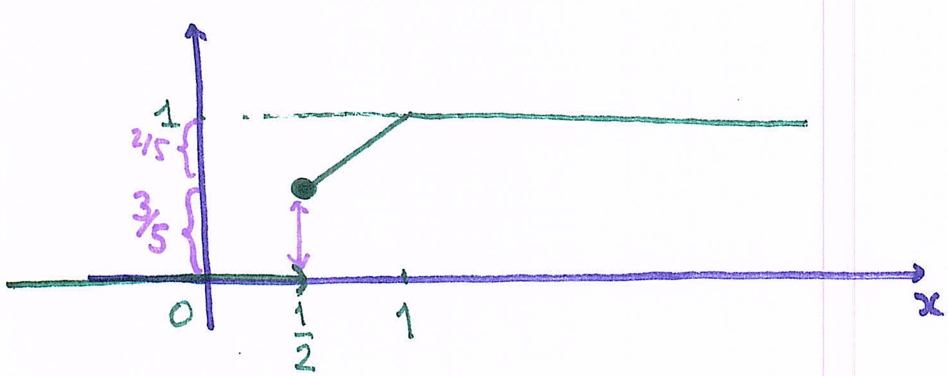
e.g.,



discrete, since the cdf is a step function (w/ @ most countably many jumps)



continuous dist'n w/ bounded support



a mixed dist'n.

M339J : January 27th, 2020.

* **175.** An insurance company's annual profit is normally distributed with mean 100 and variance 400.

Let Z be normally distributed with mean 0 and variance 1 and let F be the cumulative distribution function of Z.

Determine the probability that the company's profit in a year is at most 60, given that the profit in the year is positive.

conditional probability

(A) $1 - F(2)$

(B) $F(2)/F(5)$

(C) $[1 - F(2)]/F(5)$

(D) $[F(0.25) - F(0.1)]/F(0.25)$

(E) $[F(5) - F(2)]/F(5)$

176. In a group of health insurance policyholders, 20% have high blood pressure and 30% have high cholesterol. Of the policyholders with high blood pressure, 25% have high cholesterol.

A policyholder is randomly selected from the group.

Calculate the probability that a policyholder has high blood pressure, given that the policyholder has high cholesterol.

(A) 1/6
(B) 1/5
(C) 1/4
(D) 2/3
(E) 5/6

2.

$\rightarrow$: X ... the total annual profit

$X \sim N(\text{mean} = 100, \text{var} = 400)$

$$\tilde{p} := \mathbb{P}[X \leq 60 \mid X > 0] \underset{\substack{\uparrow \\ \text{def'n.}}}{=} \frac{\mathbb{P}[0 < X \leq 60]}{\mathbb{P}[X > 0]}$$

We can rewrite $\boxed{X = 100 + 20 \cdot Z}$

i.e., $Z = \frac{X - 100}{20}$

$$\Rightarrow \tilde{p} = \frac{\mathbb{P}\left[\frac{0-100}{20} < Z \leq \frac{60-100}{20}\right]}{\mathbb{P}\left[Z > \frac{0-100}{20}\right]} =$$

$$= \frac{\mathbb{P}[-5 < Z \leq -2]}{\mathbb{P}[-5 < Z]} = \frac{\mathbb{P}[Z \leq -2] - \mathbb{P}[Z \leq -5]}{1 - \mathbb{P}[Z \leq -5]}$$

$$= \frac{F(-2) - F(-5)}{1 - F(-5)}$$

In the std normal case: $F(-z) = 1 - F(z)$

$$\Rightarrow \tilde{p} = \frac{F(5) - F(2)}{F(5)} \Rightarrow (E)$$

Exponential Dist'n

Any exponential random variable X w/ **parameter** θ has the density

$$f_X(x) = \frac{1}{\theta} e^{-\frac{x}{\theta}}$$

for $x > 0$

(and 0 otherwise)

So: its support is $\mathbb{R}_+ = \{ x \in \mathbb{R} : x > 0 \}$

Its cdf is

$$F_X(x) = \mathbb{P}[X \le x] = \int_0^x f_X(u)\, du$$

$$= \int_0^x \frac{1}{\theta} e^{-\frac{u}{\theta}}\, du =$$

$$= \frac{1}{\theta}(-\theta) e^{-\frac{u}{\theta}} \Big|_{u=0}^{x} =$$

$$\boxed{F_X(x) = 1 - e^{-\frac{x}{\theta}}}$$

$\Rightarrow$ The survival f'tion: $S_X(x) = e^{-\frac{x}{\theta}}$

The hazard rate is:

$$h_X(x) = \frac{1}{\theta}$$

The mode is zero.

35. This question duplicates Question 34 and has been deleted.

∗ 36. A group insurance policy covers the medical claims of the employees of a small company. The value, V, of the claims made in one year is described by

$$V = 100{,}000Y$$

where Y is a random variable with density function

$$f(y) = \begin{cases} k(1-y)^4, & 0 < y < 1 \\ 0, & \text{otherwise} \end{cases}$$

where k is a constant.

Calculate the conditional probability that V exceeds 40,000, given that V exceeds 10,000.

(A) 0.08
(B) 0.13
(C) 0.17
(D) 0.20
(E) 0.51 x

∗ 37. The lifetime of a printer costing 200 is $\underbrace{\text{exponentially distributed with mean 2}}_{\theta=2 \quad F_X(x)=1-e^{-\frac{x}{2}}}$ years. The manufacturer agrees to pay a full refund to a buyer if the printer fails during the first year following its purchase, a one-half refund if it fails during the second year, and no refund for failure after the second year.

Calculate the expected total amount of refunds from the sale of 100 printers.

(A) 6,321
(B) 7,358
(C) 7,869
(D) 10,256
(E) 12,642

$$\text{Refund} \begin{cases} 200 & \text{w/} \quad \mathbb{P}[X \leq 1] = 0.393 \\ 100 & \text{w/} \quad \mathbb{P}[1 < X \leq 2] = 0.2387 \\ 0 & \ldots \end{cases}$$

$$100\left(0.393(200) + 0.2387(100)\right) \cong 10{,}256$$

5.