M339J: March 2nd, 2020.

277. You are given:

(i) Loss payments for a group health policy follow an exponential distribution with unknown mean.

(ii) A sample of losses is:

100 200 400 800 1400 3100

Using the delta method, calculate the approximation of the variance of the maximum likelihood estimator of $S(1500)$.

(A) 0.019

(B) 0.025

(C) 0.032

(D) 0.039

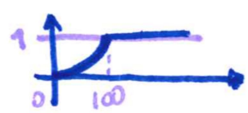
(E) 0.045

278. DELETED

1.

✗ **279.** Loss amounts have the distribution function

$$F(x) = \begin{cases} (x/100)^2, & 0 \le x \le 100 \\ 1, & x > 100 \end{cases}$$



$\alpha = 0.80$

An insurance pays 80% of the amount of the loss in excess of an ordinary deductible of 20, subject to a maximum payment of 60 per loss.

$d = 20$

$0.8(u - 20) = 60 \Rightarrow u = 95$

Calculate the conditional expected claim payment, given that a payment has been made.

(A) 37

(B) 39

(C) 43

(D) 47

(E) 49

$$\mathbb{E}[Y^P] = \frac{\mathbb{E}[Y^L]}{S_X(d)}$$

$$\cdot\ \mathbb{E}[Y^L] = \alpha\left(\mathbb{E}[X \wedge \overset{95}{u}] - \mathbb{E}[X \wedge \overset{20}{d}]\right)$$

For any constant $c \in (0, 100)$:

$$\mathbb{E}[X \wedge c] = \int_0^c S_X(x)\,dx =$$

$$= \int_0^c \left(1 - \frac{x^2}{10^4}\right) dx$$

$$= x - \frac{1}{10^4} \cdot \frac{x^3}{3} \Big]_{x=0}^{3}$$

$$= c - \frac{1}{10^4} \cdot \frac{c^3}{3}$$

$$\Rightarrow \mathbb{E}[Y^L] = 0.8\left(95 - \frac{1}{10^4} \cdot \frac{95^3}{3} - \left(20 - \frac{1}{10^4} \cdot \frac{20^3}{3}\right)\right)$$

$$= \cdots = 37.35$$

$$\Rightarrow \mathbb{E}[Y^P] = \frac{37.35}{1 - \left(\frac{20}{100}\right)^2} = \frac{37.35}{0.96} = 38.91 \Rightarrow \text{(B)}$$

THM.

- $\mathbb{E}[(Y^L)^2] = \alpha^2(1+r)^2\Big(\mathbb{E}\big[(X\wedge \frac{u}{1+r})^2\big] - \mathbb{E}\big[(X\wedge \frac{d}{1+r})^2\big]$
$$-2\cdot\left(\frac{d}{1+r}\right)\mathbb{E}\left[X\wedge \frac{u}{1+r}\right]$$
$$+2\left(\frac{d}{1+r}\right)\mathbb{E}\left[X\wedge \frac{d}{1+r}\right]\Big)$$

- $\mathbb{E}[(Y^P)^2] = \dfrac{\mathbb{E}[(Y^L)^2]}{S_X(\frac{d}{1+r})}$

Reading Assignment: Section 6.1.

Section 6.2. The Poisson Dist'n.

It's customary to use

$$N \sim \text{Poisson}(\lambda)$$

Q: What's the support?

$$\mathbb{N}_0 = \{0, 1, 2, \dots\}$$

- The probability mass f'tion:

$$p_k = e^{-\lambda} \cdot \frac{\lambda^k}{k!} \quad , \; k = 0, 1, 2, \dots$$

- The probability generating f'tion:

$$P_N(z) = \mathbb{E}[z^N] = e^{\lambda(z-1)}$$

- $\mathbb{E}[N] = Var[N] = \lambda$

Bayes' Theorem:

$$\mathbb{P}[AS \mid 3\text{ colds}] = \frac{\mathbb{P}[AS \text{ and } 3\text{ colds}]}{\mathbb{P}[3\text{ colds}]}$$

$$= \frac{0.1 \cdot e^{-4} \cdot \frac{4^3}{\cancel{6}}}{0.3 e^{-3} \cdot \frac{3^3}{\cancel{6}} + 0.6 \cdot e^{-1} \cdot \frac{1^3}{\cancel{6}} + 0.1 e^{-4} \cdot \frac{4^3}{\cancel{6}}}$$

$$= 0.158 \approx 0.16 \Rightarrow (B)$$

* **170.** In a certain town the number of common colds an individual will get in a year follows a Poisson distribution that depends on the individual's age and smoking status. The distribution of the population and the mean number of colds are as follows:

	Proportion of population	Mean number of colds
Children	0.30	3
Adult Non-Smokers	0.60	1
Adult Smokers	0.10	4

Calculate the conditional probability that a person with exactly 3 common colds in a year is an adult smoker.

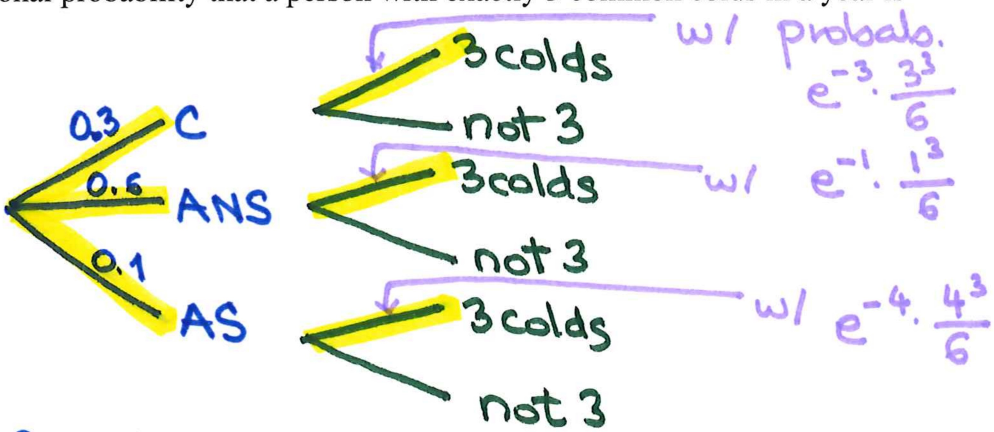
(A) 0.12

(B) 0.16

(C) 0.20

(D) 0.24

(E) 0.28



Our question: $\mathbb{P}[\text{AS} \mid 3\text{colds}] = ?$

171. For aggregate losses, S:

(i) The number of losses has a negative binomial distribution with mean 3 and variance 3.6.

(ii) The common distribution of the independent individual loss amounts is uniform from 0 to 20.

Calculate the 95^{th} percentile of the distribution of S as approximated by the normal distribution.

(A) 61

(B) 63

(C) 65

(D) 67

(E) 69

130. Bob is a carnival operator of a game in which a player receives a prize worth $W = 2^N$ if the player has N successes, $N = 0, 1, 2, 3, \ldots$ Bob models the probability of success for a player as follows:

(i) N has a Poisson distribution with mean Λ.

(ii) Λ has a uniform distribution on the interval (0, 4).

} A MIXING DIST'N :)

Calculate $E[W]$.

(A) 5

(B) 7

(C) 9

(D) 11

(E) 13

$N | \Lambda = \lambda \sim \text{Poisson}(\lambda)$
$\Lambda \sim U(0,4)$

$$\mathbb{E}[W] = \mathbb{E}[2^N] = \mathbb{E}[\underbrace{\mathbb{E}[2^N | \Lambda]}_{\text{use the p.g.f. expression } e^{\Lambda(2-1)} = e^{\Lambda}}]$$

$$\Rightarrow \mathbb{E}[W] = \mathbb{E}[e^{\Lambda}] \underset{\Lambda \sim U(0,4)}{=} \int_0^4 e^{\lambda} \cdot \frac{1}{4}\, d\lambda = \frac{1}{4}(e^4 - 1)$$

$= 13.4 \Rightarrow$ (E)

131. DELETED

132. DELETED

(7.)

THM. Let $N_1, N_2, \dots, N_\ell$ be independent Poisson random variables w/ parameters $\lambda_1, \lambda_2, \dots, \lambda_\ell$, resp.
Set $N := N_1 + N_2 + \dots + N_\ell$

Then: $N \sim \text{Poisson}(\lambda = \lambda_1 + \dots + \lambda_\ell)$

[THINNING]

THM. Let $N \sim \text{Poisson}(\lambda)$ be a counting random variable for some events of interest. Suppose that independently of N, each event falls into a particular category $i = 1, \dots, m$ w/ probability p_i $(i = 1 \dots m)$.
Let N_i be the # of events of type i, for all $i = 1 \dots m$.

Then:
- $N_i \sim \text{Poisson}(\lambda \cdot p_i)$ for all i
- $(N_1, N_2, \dots, N_m)$ are independent random variables

111. The number of accidents follows a Poisson distribution with mean 12. Each accident generates 1, 2, or 3 claimants with probabilities 1/2, 1/3, and 1/6, respectively.

Calculate the variance of the total number of claimants.

(A) 20

(B) 25

(C) 30

(D) 35

(E) 40

112. In a clinic, physicians volunteer their time on a daily basis to provide care to those who are not eligible to obtain care otherwise. The number of physicians who volunteer in any day is uniformly distributed on the integers 1 through 5. The number of patients that can be served by a given physician has a Poisson distribution with mean 30.

Determine the probability that 120 or more patients can be served in a day at the clinic, using the normal approximation with continuity correction.

(A) $1-\Phi(0.68)$

(B) $1-\Phi(0.72)$

(C) $1-\Phi(0.93)$

(D) $1-\Phi(3.13)$

(E) $1-\Phi(3.16)$

9.

THINNING

EVENT $\xrightarrow{1/2}$ 1 CL : $N_1 \sim \text{Poisson}\left(\lambda_1 = \frac{1}{2} \cdot 12 = 6\right)$

EVENT $\xrightarrow{1/3}$ 2 CL : $N_2 \sim \text{Poisson}\left(\lambda_2 = \frac{1}{3} \cdot 12 = 4\right)$

EVENT $\xrightarrow{1/6}$ 3 CL : $N_3 \sim \text{Poisson}\left(\lambda_3 = \frac{1}{6} \cdot 12 = 2\right)$

total # of all events : $N \sim \text{Poisson}(\lambda = 12)$

N_i ... # of accidents w/ i claimants for $i = 1, 2, 3$

S ... total # of CLAIMANTS: $S = 1 \cdot N_1 + 2 \cdot N_2 + 3 \cdot N_3$

$\text{Var}[S] = ?$

$\text{Var}[S] = \text{Var}[N_1 + 2N_2 + 3N_3] =$

By our thinning THM, N_1, N_2, N_3 are independent

$$= \text{Var}[N_1] + 4\text{Var}[N_2] + 9 \cdot \text{Var}[N_3]$$
$$= 6 + 4 \cdot 4 + 9 \cdot 2$$
$$= 40 \quad \blacksquare$$