Name:

M339J: Probability models
University of Texas at Austin
Sample In-Term Exam I
Instructor: Milica Čudina

Notes: This is a closed book and closed notes exam. The maximal score on this exam is 50 points.
Time: 50 minutes

TRUE/FALSE

1 (2)	TRUE	FALSE
2 (2)	TRUE	FALSE
3 (2)	TRUE	FALSE
4 (1)	TRUE	FALSE
5 (2)	TRUE	FALSE

MULTIPLE CHOICE

1 (5)	a	b	c	d	e
2 (5)	a	b	c	d	e
3 (5)	a	b	c	d	e
4 (5)	a	b	c	d	e

1.1. **DEFINITION.**

Problem 1.1. Provide the definition of a *cumulative distribution function* of a random variable X.

1.2. **TRUE/FALSE QUESTIONS.** *Please, note your final answer on the front page of this exam.*

Problem 1.2. (2 pts) Let X be an exponential random variable. Then, its mean and its standard deviation are equal. *True or false?*

Problem 1.3. (2 pts) If the mean excess loss function is increasing, then the corresponding distribution has a heavy tail. *True or false?*

Problem 1.4. (2 points) Let N be an $\mathbb{N}_0$−valued random variable with the probability generating function P_N. Then,

$$\mathbb{E}[N] = P_N'(0).$$

True or false?

Problem 1.5. (2 points) For a random variable X and for a positive constant d, in our usual notation, we have

$$\mathbb{E}[X] = e_X(d)S_X(d) + \mathbb{E}[X \wedge d]\,. \tag{1.1}$$

True or false?

Problem 1.6. (2 pts) Let X have the loglogistic distribution. Then, the random variable $X' = 1/X$ also has the loglogistic distribution. *True or false?*

1.3. **Free-response problems.** Please, explain carefully all your statements and assumptions. Numerical results or single-word answers without an explanation (even if they're correct) are worth 0 points.

Problem 1.7. (10 points) A population of insureds consists of three types of people: α, β and γ. There is an equal number of Type α and Type β people in the population. The number of Type γ people is equal to the total number of the remaining two types of people. The probability that a Type α person makes at least one claim in a year is $1/5$. The probability that a Type β person makes at least one claim in a year is $2/5$. The probability that a Type γ person makes at least one claim in a year is $3/5$.

At the end of the year, a person is chosen at random from the population. It is observed that that person had at least one claim. What is the probability that the person was of Type β?

Problem 1.8. (15 points) Losses X follow a Pareto distribution with parameters $\alpha > 1$ and θ unspecified. For a positive constant c, determine the ratio of the mean excess loss function evaluated at $c\theta$ to the mean excess loss function evaluated at θ.

Problem 1.9. (10 points) Let X have the exponential distribution with mean 4 and define $Y = \sqrt{X}$. Find $f_Y(3)$.

1.4. **MULTIPLE CHOICE QUESTIONS.** *Please, note your final answers on the front page of this exam.*

Problem 1.10. (5 pts) *Source: Prof. Jim Daniel (personal communication).*
The ground-up loss X is modeled by an Exponential distribution with mean \$500. There is an ordinary deductible of $d = 100$. What can you say about the expected value of the per-loss random variable?

(a) It is less than 100.
(b) It is more than 100, but less than 250.
(c) It is more than 250, but less than 375.
(d) It is more than 375, but less than 500.
(e) None of the above

Problem 1.11. (5 pts) Let N be Poisson with variance 5 and let Z be Bernoulli with probability of success equal to 1/5. Assume that N and Z are independent. Let $\pi = \mathbb{P}[N + Z = 4]$. Then, we have that

(a) $0 \leq p < 1/4$
(b) $1/4 \leq p < 1/2$
(c) $1/2 \leq p < 5/8$
(d) $5/8 \leq p < 9/10$
(e) None of the above

Problem 1.12. (5 pts) Let X be exponential with variance 225. Let $a = \mathbb{E}[|20 - X|]$. Then,

(a) $0 \leq a < 50$
(b) $50 \leq a < 150$
(c) $150 \leq a < 325$
(d) $325 \leq a < 550$
(e) None of the above.

Problem 1.13. (5 pts) Let X be exponential with mean 4 and let x_* denote its 90^{th} percentile. Then,

(a) $0 \leq x_* < 3$
(b) $3 \leq x_* < 5$
(c) $5 \leq x_* < 8$
(d) $8 \leq x_* < 10$
(e) None of the above.

Problem 1.14. (5 points) A continuous random variable X has the *probability density function* of the following form

$$f_X(x) = \kappa x^{-5} \quad \text{for } x > 1 \tag{1.2}$$

where κ is a positive constant. Let x_* denote its 95^{th} percentile. Then,

(a) $0 \leq x_* < 1$
(b) $1 \leq x_* < 2$
(c) $2 \leq x_* < 3$
(d) $3 \leq x_* < 4$
(e) None of the above.

Problem 1.15. (5 pts)Let X be the ground-up loss random variable. Assume that X has the exponential distribution with mean $5,000$.
Let B denote the expected payment per loss on behalf of an insurer which wrote a policy with a deductible of $1,500$ and with no maximum policy payment. Then,

(a) $B \approx 1,700$
(b) $B \approx 2,700$
(c) $B \approx 3,700$
(d) $B \approx 4,700$
(e) None of the above