Name:

M339J Probability models with actuarial applications
University of Texas at Austin
Sample In-Term Exam III
Instructor: Milica Čudina

Notes: This is a closed book and closed notes exam.
Time: 50 minutes

3.1. **TRUE/FALSE QUESTIONS.**

Problem 3.1. (2 points) With our usual assumptions in place, assume that the claim count follows the Poisson distribution with parameter $\lambda = 10$.
The severities of individual claims are such that on average half of the claims are expected to have claim amounts equal to 50 and the other half are expected to have the claim amounts equal to 100. Then, the total losses have the expected value 750. *True or false?*

Problem 3.2. (2 points) In the compound aggregate loss model, with our usual notation and with the usual assumptions in place, we have that the claim count distribution is binomial with $m = 40$ and $q = 1/4$, and that the p.m.f. of the individual losses is

$$p_X(10) = p_X(20) = 0.3, p_X(50) = 0.4.$$

Then, $\mathbb{E}[S] \leq 290$. *True or false?*

3.2. **Free-response problems.** *Please, explain carefully all your statements and assumptions. Numerical results or single-word answers without an explanation (even if they're correct) are worth 0 points.*

Problem 3.3. (10 points) In the compound model for aggregate claims, let the frequency random variable N have the Poisson distribution with mean 1.

Let the common distribution of the i.i.d. severity random variables $\{X_j; j = 1, 2, \dots\}$ be given by the following p.m.f.

$$p_X(100) = 1/2, p_X(200) = 3/10, p_X(300) = 1/5.$$

Let our usual assumptions hold, i.e., let N be independent of $\{X_j; j = 1, 2, \dots\}$.
Define the aggregate loss as $S = \sum_{j=1}^{N} X_j$.
Find the expected value of the **policyholder's** payment for a stop-loss insurance policy with an ordinary deductible of 200, i.e., calculate $\mathbb{E}[S \wedge 200]$.

Problem 3.4. (10 points) In the compound model for aggregate claims, let the frequency random variable N have the Poisson distribution with mean 20.

Moreover, let the common distribution of the i.i.d. severity random variables $\{X_j; j = 1, 2, \dots\}$ be the Gamma distribution with parameters $\alpha = 2.5$ and $\theta = 3,000$.

Let our usual assumptions hold, i.e., let N be independent of $\{X_j; j = 1, 2, \dots\}$.
Define the aggregate loss as $S = \sum_{j=1}^{N} X_j$.

Using the normal approximation, find the amount of the premium π such that the total aggregate losses S exceed the premium with the probability of at most 1%.

Problem 3.5. (10 points) The frequency random variable N is assumed to have a Poisson distribution with a mean of 2. Individual claim severity random variable X has the following probability mass function

$$p_X(100) = 0.6, \quad p_X(200) = 0.3, \quad p_X(300) = 0.1.$$

Let the above be the common distribution of the i.i.d. severity random variables $\{X_j; j = 1, 2, \dots\}$, and Let our usual assumptions hold, i.e., let N be independent of $\{X_j; j = 1, 2, \dots\}$. Define the aggregate loss as $S = \sum_{j=1}^{N} X_j$. Calculate the probability that S is exactly equal to 3.

Problem 3.6. (6 points) In the compound model for aggregate claims, let the frequency random variable N have the probability (mass) function

$$p_N(0) = 0.4, p_N(1) = 0.3, p_N(2) = 0.2, p_N(4) = 0.1.$$

Moreover, let the common distribution of the i.i.d. severity random variables $\{X_j; j = 1, 2, \dots\}$ be given by the probability (mass) function $p_X(1) = 0.3$ and $p_X(2) = 0.7$.
Let our usual assumptions hold, i.e., let N be independent of $\{X_j; j = 1, 2, \dots\}$.
Define the aggregate loss as $S = \sum_{j=1}^{N} X_j$.
Calculate $\mathbb{E}[(S-2)_+]$.

3.3. **MULTIPLE CHOICE QUESTIONS.**

Problem 3.7. (5 points) Consider the following individual observed values:

$$5, 8, 10$$

of a random variable X whose distribution function is given by $F_X(x) = 1 - (1/x)^p$ for $x > 1$ and an unknown parameter p. Let $\hat{p}$ denote the Maximum Likelihood Estimate of p based on the above observed values. Then,

(a) $\hat{p} \approx \frac{3}{2}$
(b) $\hat{p} \approx \frac{3\ln(20)}{2}$
(c) $\hat{p} \approx \frac{3\ln(20)}{4}$
(d) $\hat{p} \approx \frac{3}{2\ln(20)}$
(e) None of the above

Problem 3.8. (5 points) In the compound model for aggregate claims, let the frequency random variable N have the Poisson distribution with mean 3. Moreover, let the common distribution of the i.i.d. severity random variables $\{X_j; j = 1, 2, \dots\}$ be the two-parameter Pareto with parameters $\alpha = 3$ and $\theta = 5$. Let our usual assumptions hold, i.e., let N be independent of $\{X_j; j = 1, 2, \dots\}$.
Define the aggregate loss as $S = \sum_{j=1}^{N} X_j$.
Then,

(a) $0 \leq Var[S] < 25$
(b) $25 \leq Var[S] < 50$
(c) $50 \leq Var[S] < 65$
(d) $65 \leq Var[S] < 80$
(e) None of the above

Problem 3.9. (5 points) In the compound model for aggregate claims, let the frequency random variable N have the geometric distribution with mean 4. Moreover, let the individual losses be always equal to 50. Define the aggregate loss as $S = \sum_{j=1}^{N} X_j$. Then,

(a) $\mathbb{E}[(S - 100)_+] \approx 64$
(b) $\mathbb{E}[(S - 100)_+] \approx 128$
(c) $\mathbb{E}[(S - 100)_+] \approx 192$
(d) $\mathbb{E}[(S - 100)_+] \approx 256$
(e) None of the above

Problem 3.10. (5 points) Let S be the aggregate claims random variable. You are given the following:
(i) $\mathbb{E}[(S - 100)_+] = 15$,
(ii) $\mathbb{E}[(S - 120)_+] = 10$,
(iii) $\mathbb{P}[80 < S \leq 120] = 0$.
Find $\mathbb{E}[(S - 105)_+]$.

(a) $\mathbb{E}[(S - 105)_+] \approx 10.75$
(b) $\mathbb{E}[(S - 105)_+] \approx 12.75$
(c) $\mathbb{E}[(S - 105)_+] \approx 13.75$
(d) $\mathbb{E}[(S - 105)_+] \approx 15.75$
(e) None of the above

Problem 3.11. (5 points) Consider the following individual observed values

$$5, 10$$

and the one right censored value 8 of a random variable X whose distribution function is given by $F_X(x) = 1 - (1/x)^p$ for $x > 1$ and an unknown parameter p. Let $\hat{p}$ denote the Maximum Likelihood Estimate of p based on the above observed values. Then,

(a) $\hat{p} \approx \frac{1}{\ln(20)}$
(b) $\hat{p} \approx \frac{3\ln(20)}{2}$
(c) $\hat{p} \approx \frac{3}{4\ln(20)}$
(d) $\hat{p} \approx \frac{3}{2\ln(20)}$
(e) None of the above

Problem 3.12. The number of claims in a particular time-period, denoted by N, has a geometric distribution with mean 3. The amount of each claim X is uniform on $\{1,2,3,4\}$. The number of claims and the claim amount are independent. Let S be the aggregate claim amount in the period. Calculate $F_S(2)$.

(a) 57/1024
(b) 361/1024
(c) 551/1024
(d) 613/1024
(e) None of the above.

Problem 3.13. You fit a distribution with the following density function:

$$f_X(x) = (p+1)x^p, \quad 0 < x < 1, p > -1.$$

As usual, your observations are denoted by

$$x_1, x_2, \ldots, x_n.$$

What is the expression for the maximum likelihood estimate of the parameter p?

(a) $-\frac{n}{\sum_{i=1}^{n} \ln(x_i)} - 1$
(b) $-\frac{1}{n}\sum_{i=1}^{n} \ln(x_i)$
(c) $\frac{1}{n}\sum_{i=1}^{n} x_i$
(d) $\frac{\sum_{i=1}^{n} \ln(x_i)}{n} - 1$
(e) None of the above.

Problem 3.14. You have observed the following three loss amounts:

$$190, \quad 90, \quad 60$$

Four other loss amounts are known to be less than or equal to 60. Losses follow an inverse exponential distribution. Calculate the maximum likelihood estimate of the population mode based on the above data.

(a) 10.125

(b) 12.378

(c) 15.044

(d) 20.232

(e) None of the above.

Problem 3.15. *Source: Sample STAM Problem#196.*
You are given the following 10 bodily injury losses (before the deductible is applied):

Loss amount	Number of losses	Policy limit
200	3	500
400	4	800
> 800	3	800

Past experience indicates that these losses follow a two-parameter Pareto distribution with parameters α unknown and $\theta = 1,000$. Calculate the maximum likelihood estimate of α.

(a) 1.9145

(b) 2.307

(c) 2.853

(d) 3.089

(e) None of the above.

Problem 3.16. *Source: Sample STAM Problem #152.*
You are given the following information:

(i) A sample of losses is: $400, 600, 700, 900, 1000$
(ii) No information is available about losses of 300 or less.
(iii) Losses are assumed to follow an exponential distribution with mean θ.

Calculate the maximum likelihood estimate of θ based on the above data.

(a) 420

(b) 520

(c) 720

(d) 920

(e) None of the above.