

Name:

*M339J/M389J: Probability Models with Actuarial Applications*

The University of Texas at Austin

**Sample Problems for In-Term Exam II**

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**Notes:** This is a closed book and closed notes exam. The maximal score on this exam is 50 points.

**Time:** 50 minutes

**TRUE/FALSE**

1 (2)	TRUE	FALSE
2 (2)	TRUE	FALSE
3 (2)	TRUE	FALSE
4 (2)	TRUE	FALSE
5 (2)	TRUE	FALSE

**MULTIPLE CHOICE**

1 (5)	a	b	c	d	e
2 (5)	a	b	c	d	e
3 (5)	a	b	c	d	e
4 (5)	a	b	c	d	e
5 (5)	a	b	c	d	e

Part I. **TRUE/FALSE QUESTIONS**

- (2 pts) Let  $X_1$  and  $X_2$  be independent, exponentially distributed random variables. Define  $X = X_1 + X_2$ . Then,  $X$  is also exponential. *True or false?*
- (2 pts) Let  $N$  have a mixture distribution with the mixing variable  $\Lambda$ . More precisely, let

$$N | \Lambda = \lambda \sim \text{Poisson}(\lambda)$$

with  $\Lambda \sim \text{Gamma}(\alpha, \theta)$ . Then,

$$N \sim \text{NegBin}(\alpha, \theta).$$

*True or false?*

- (2 pts) Any negative binomial distribution has the memoryless property. *True or false?*

Part II. Please, explain carefully all your statements and assumptions. Numerical results or single-word answers without an explanation (even if they're correct) are worth 0 points.

- (10 points) Let  $X \sim \text{Pareto}(\alpha = 3, \theta = 3000)$ . Assume that there is a deductible of  $d = 5000$ . Find the loss elimination ratio.
- (10 pts) Assume that conditional on the random variable  $\Lambda$ ,  $N$  has the Poisson distribution with variance  $\Lambda$ . Let  $\Lambda \sim U(0, 2)$ . Find the (unconditional) mean and variance of  $N$ .
- (10 points) Let the ground-up loss  $X$  be exponentially distributed with mean \$800. An insurance policy has an ordinary deductible of \$100 and the maximum amount payable per loss of \$2500. Find the expected value of the amount paid (by the insurance company) per positive payment.
- (10 pts) Losses in year  $y$  follow a two-parameter Pareto distribution with parameters  $\alpha = 4$  and  $\theta = 10$ . Losses in year  $y + 1$  are uniformly 10% higher than those in year  $y$ . An insurance covers each loss subject to a deductible  $d = 20$ . Calculate the **loss elimination ratio** for year  $y + 1$ .
- (10 points) Assume that the severity random variable  $X$  is uniform on the interval  $(0, 1000)$ . There is an insurance policy to cover this loss. The insurance policy has a deductible of 200 per loss and the maximum amount payable by the insurer per loss of 700.

Calculate the expected value of the per payment random variable  $Y^P$  under this policy.

7. (20 points) Assume that the severity random variable  $X$  is exponentially distributed with mean 1400. There is an insurance policy to cover this loss. This insurance policy has
- (1) a deductible of 200 per loss,
  - (2) the coinsurance factor of  $\alpha = 0.25$ , and
  - (3) the maximum amount payable by the insurer per loss of 700.

Calculate the expected value of the per payment random variable  $Y^P$  under this policy.

### Part III. MULTIPLE CHOICE QUESTIONS

1. (5 pts) Let us denote the claim count r.v. by  $N$ . We are given that  $N$  is a mixture random variable such that

$$N | \Lambda = \lambda \sim \text{Poisson}(\lambda)$$

while  $\Lambda$  is Gamma distributed with mean 2 and variance equal to 4. Then,

- (a)  $F_N(1) \leq 0.1$
  - (b)  $0.1 < F_N(1) \leq 0.2$
  - (c)  $0.2 < F_N(1) \leq 0.3$
  - (d)  $0.3 < F_N(1) \leq 0.5$
  - (e) None of the above
2. (5 pts) Let  $X$  be the ground-up loss random variable. Assume that  $X$  has the two-parameter Pareto distribution with  $\theta = 4,000$  and  $\alpha = 3$ . Let  $B$  denote the expected payment per loss on behalf of an insurer which wrote a policy with no deductible and with a policy limit of 5,000. Then,
- (a)  $B \approx 1,000$
  - (b)  $B \approx 1,200$
  - (c)  $B \approx 1,400$
  - (d)  $B \approx 1,600$
  - (e) None of the above

3. (5 pts) Let  $X$  be the ground-up loss random variable. Assume that  $X$  has the exponential distribution with mean 1,000. Let  $B$  denote the expected payment per loss on behalf of an insurer who wrote a policy with a deductible of 1,500 and with the **maximum payment by the insurer** equal to 2,500. Then,
- (a)  $B \approx 714$
  - (b)  $B \approx 816$
  - (c)  $B \approx 918$
  - (d)  $B \approx 1020$
  - (e) None of the above
4. (5 pts) Assume that  $X$  is a mixture distribution with the mixing variable  $\Lambda$ . More precisely, let  $X | \Lambda = \lambda$  be uniform on the interval  $(0, \lambda)$  and let  $\Lambda$  be uniform on an interval of the form  $(0, c)$  for some constant  $c$  with mean 5. Let  $\pi = \mathbb{P}[X \leq 3]$ . Then, we have that
- (a)  $0 \leq p < 1/4$
  - (b)  $1/4 \leq p < 1/2$
  - (c)  $1/2 \leq p < 5/8$
  - (d)  $5/8 \leq p < 9/10$
  - (e) None of the above

**Problem 0.1.** (5 points) A model for the arrival time  $T$  for a particular event is initially an exponential distribution with mean 2 years. Upon reconsideration, this distribution is replaced with a spliced model whose density function:

- (i) is uniform over  $[0, 1]$ ,
- (ii) is proportional to the initial modeled density function after 1 year,
- (iii) is continuous.

Calculate the probability of failure in the first 3 year under the revised distribution.

- (a)  $1/4$
- (b)  $1/3$
- (c)  $1/2$
- (d)  $3/5$
- (e) None of the above.

**Problem 0.2.** (5 points) Let the ground-up loss  $X$  be modeled by a two-parameter Pareto distribution with parameters  $\alpha = 2$  and  $\theta = 200$ . For an insurance policy on the above loss, there is a franchise deductible of 200. Find the expected value of the per payment random variable.

- (a) 200
- (b) 400
- (c) 600
- (d) 800
- (e) None of the above.

**Problem 0.3.** (5 points) *Source: Sample P exam, Problem #262.*

The number of traffic accidents occurring on any given day in Coralville is Poisson distributed with mean 5. The probability that any such accident involves an uninsured driver is 0.25, independent of all other such accidents. Calculate the probability that on a given day in Coralville there are no traffic accidents that involve an uninsured driver.

- (a) 0.007
- (b) 0.010
- (c) 0.124
- (d) 0.237
- (e) 0.287

**Problem 0.4.** (5 points) A group dental policy has a negative binomial claim count distribution with mean 20 and variance 100. What is the probability that there is at most 1 claim?

- (a) 0.0016
- (b) 0.0024
- (c) 0.0032
- (d) 0.0048
- (e) None of the above.

**Problem 0.5.** (5 points) Let the number of typos in the new edition of the "*Lord of the Rings*" trilogy be denoted by  $N$  and modeled using the Poisson distribution with mean 80. Some typos involve elves' names and others do not involve the elves' names. The number of typos and the type of the typo are assumed to be independent.

Assume that the probability that an observed typo involves an elf's name equals  $1/5$ .

Find the probability that the number of typos involving elves' names is 2, given that the **total** number of typos in the trilogy equals 5.

- (a) 0.0406
- (b) 0.0842
- (c) 0.1632
- (d) 0.2048
- (e) None of the above.

**Problem 0.6.** (8 points) Let  $N$  be an  $\mathbb{N}_0$ -valued random variable from the  $(a, b, 0)$  class. You are given that

$$p_0 = 0.049787, \quad p_1 = 0.149361, \quad \text{and} \quad p_2 = 0.224042.$$

Find  $p_6$ .

- (a) 0.04
- (b) 0.05
- (c) 0.06
- (d) 0.07
- (e) None of the above.