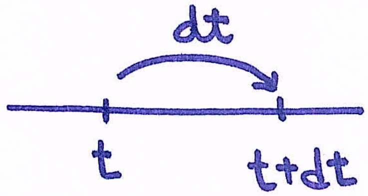
Delta·Gamma·Theta Approximation.

W: April 1st, 2019.



$$
\begin{aligned}
v(s+ds,\, t+dt) \cong\ & v(s,t) \\
& + \Delta(s,t)\, ds \\
& + \frac{1}{2}\Gamma(s,t)(ds)^2 \\
& + \Theta(s,t)\, dt
\end{aligned}
$$

- If we ignore the Θ-term, i.e., the dt·term, we end up w/ the Delta·Gamma Approximation.

19. Assume that the Black-Scholes framework holds. The price of a nondividend-paying stock is \$30.00. The price of a put option on this stock is \$4.00.

You are given:

(i) $\Delta = -0.28$

(ii) $\Gamma = 0.10$

Using the delta-gamma approximation, determine the price of the put option if the stock price changes to \$31.50.

$\Rightarrow ds = 31.50 - 30 = 1.50$

(A) \$3.40

(B) \$3.50

(C) \$3.60

(D) \$3.70

(E) \$3.80

$$v_p(S(dt), dt) \cong v_p(S(0), 0) + \Delta_p(S(0), 0)\, ds + \frac{1}{2}\Gamma_p(S(0), 0)(ds)^2$$

$$= 4 + (-0.28)\cdot(1.50) + \frac{1}{2}(0.10)(1.50)^2$$

$$= \dots \cong 3.70 \quad \Rightarrow (D)$$

2.

STOP

20. Assume that the Black-Scholes framework holds. Consider an option on a stock.

You are given the following information at time 0:

(i) The stock price is $S(0)$, which is greater than 80. $S(0) > 80$

(ii) The option price is 2.34. $\Rightarrow$ $v(S(0),0) = 2.34$

(iii) The option delta is –0.181. $\Rightarrow$ $\Delta(S(0),0) = -0.181$

(iv) The option gamma is 0.035. $\Rightarrow$ $\Gamma(S(0),0) = 0.035$

$S(dt) = 86$

The stock price changes to 86.00. Using the delta-gamma approximation, you find that the option price changes to 2.21.

$$\approx v(S(dt), dt) \cong v(S(0),0) + \Delta(S(0),0)\, ds + \frac{1}{2}\Gamma(S(0),0)(ds)^2$$

Determine $S(0)$.

(A) 84.80 $\Rightarrow ds = 1.20$

(B) 85.00 $\Rightarrow ds = 1$

(C) 85.20

(D) 85.40

(E) 85.80

$$\Rightarrow 2.21 = 2.34 + (-0.181)(ds) + \frac{1}{2}(0.035)(ds)^2$$

$$\Rightarrow 0.0175(ds)^2 - 0.181(ds) + 0.13 = 0$$

$\Rightarrow$ Solve the quadratic in (ds):

$\Rightarrow$ the possible sol'ns are: $\begin{cases} 9.57 \approx 9.56 & \times\ (i) \\ 0.77 \approx 0.78 \end{cases}$

We choose!

$\Rightarrow S(0) = 86 - 0.77 \approx 85.20 \Rightarrow (C)$

3.

Market Makers.

→ immediacy
→ inventory $\Bigg\}$ ⇒ exposure to risk

⇒ hedge

Say, a market maker writes an option whose value function is $\underline{v(s,t)}$.

⇒ Initially, they get $v(S(0),0)$.

At any time t, the value of their position is $\boxed{-v(s,t)}$

To partially hedge this exposure, they create a portfolio which does not have the first-order sensitivity to small changes in the stock price. That means that the aim is to create a Δ-neutral portfolio, i.e., a portfolio with

$\Delta_{Port}(s,t) = 0$ (w/ continuous rebalancing).

In particular, they want a portfolio w/

$$\Delta_{Port}(S(0),0) = 0$$

The most straightforward way to accomplish this is by trading in the shares of the underlying stock. Denote the # of shares in the portfolio by $N(s,t)$.

⇒ The total value of the portfolio is:

$$-v(s,t) + N(s,t)\cdot s = v_{Port}(s,t)$$

$$\Rightarrow \Delta_{Port}(s,t) = -\Delta(s,t) + N(s,t) \underset{\uparrow}{=} 0$$

Δ-neutrality!

$$\Rightarrow \boxed{N(s,t) = \Delta(s,t)}$$

Example. A market maker writes a European call.

⇒ At time t: $-v_C(s,t)$ is their liability

⇒ In the Δ-hedge, we are going to have

$$N(s,t) = \Delta_C(s,t) = \dots.$$

In particular, @ time 0:

$$N(S(0),0) = e^{-\delta \cdot T}\cdot N(d_1(S(0),0)),$$

i.e., one needs to LONG the above # of shares.

Example. A market maker writes a European put.

⇒ At time t : $-v_P(s,t)$ is the value of the unhedged position.

⇒ In the Δ-hedge, we need the # of shares of stock to be:

$$N(s,t) = \Delta_P(s,t) = \dots$$

In particular, @ time 0 :

$$N(S(0),0) = -e^{-\delta \cdot T} N(-d_1(S(0),0))$$

We need to short shares of stock to accomplish Δ-neutrality!