W: April 3rd, 2019.

9. Consider the Black-Scholes framework. A market-maker, who delta-hedges, sells a three-month at-the-money European call option on a nondividend-paying stock.

You are given:

(i) The continuously compounded risk-free interest rate is 10%.

(ii) The current stock price is 50.

(iii) The current call option delta is 0.61791.

(iv) There are 365 days in the year.

If, after one day, the market-maker has zero profit or loss, determine the stock price move over the day.

(A) 0.41

(B) 0.52

(C) 0.63

(D) 0.75

(E) 1.11

10-17. DELETED

18. A market-maker sells 1,000 1-year European gap call options, and delta-hedges the position with shares. $T=1$

You are given: $\delta=0$

(i) Each gap call option is written on 1 share of a nondividend-paying stock.

(ii) The current price of the stock is 100. $S(0)=100$

(iii) The stock's volatility is 100%. $\sigma=1$

(iv) Each gap call option has a strike price of 130. $K_s=130$

(v) Each gap call option has a payment trigger of 100. $K_t=100$

(vi) The risk-free interest rate is 0%. $r=0$

Under the Black-Scholes framework, determine the initial number of shares in the delta-hedge.

1.

(A) 586
(B) 594
(C) 684
(D) 692
(E) 797

19. Consider a *forward start option* which, 1 year from today, will give its owner a 1-year European call option with a strike price equal to the stock price at that time.

You are given:

(i) The European call option is on a stock that pays no dividends.

(ii) The stock's volatility is 30%.

(iii) The forward price for delivery of 1 share of the stock 1 year from today is 100.

(iv) The continuously compounded risk-free interest rate is 8%.

Under the Black-Scholes framework, determine the price today of the forward start option.

(A) 11.90
(B) 13.10
(C) 14.50
(D) 15.70
(E) 16.80

2.

$\rightarrow$: We need $\Delta_{GC}(S(0),0)$ shares in the Δ-hedge.

Note: $\boxed{\text{no dividends}}$

$$\boxed{r=0}$$

$$v_{GC}(s,t) = s \cdot N(d_1(s,t)) - K_S \cdot 1 \cdot N(d_2(s,t))$$

We want to avoid the chain rule & product rule!

$$v_{GC}(s,t) = \underbrace{s \cdot N(d_1(s,t)) - K_t \cdot N(d_2(s,t))} + (K_t - K_S) \cdot N(d_2(s,t))$$

$$= v_C(s,t,\underset{\substack{\uparrow \\ \text{(strike)}}}{K_t}) + (K_t - K_S) \cdot N(d_2(s,t)) \qquad \Big/ \frac{\partial}{\partial s}$$

$\Rightarrow$

$$\Delta_{GC}(s,t) = \underbrace{\Delta_C(s,t,K_t)} + (K_t - K_S) \cdot \boxed{\frac{\partial}{\partial s} N(d_2(s,t))}$$

$$= N'(d_2(s,t)) \cdot \underbrace{\frac{\partial}{\partial s} d_2(s,t)}$$

Recall:

$$d_2(s,t) = \frac{1}{\sigma\sqrt{T-t}}\left[\ln(s) - \ln(K) + (r - \delta - \frac{\sigma^2}{2})(T-t)\right]$$

$$\Rightarrow \quad \frac{\partial}{\partial s} d_2(s,t) = \underbrace{\frac{1}{\sigma\sqrt{T-t} \cdot s}}$$

$\Rightarrow$ The initial Δ-hedge will have this number of shares of stock:

w/ strike $K_t = 100$

$$\Delta_{GC}(S(0),0) = N(d_1(S(0),0)) + (100 - 130) \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{d_2^2(S(0),0)}{2}} \cdot \frac{1}{\sigma\sqrt{T} \cdot S(0)}$$

w/ $$d_1(S(0),0) = \frac{1}{\sigma\sqrt{T}}\left[\ln\left(\frac{S(0)}{K_t}\right) + \left(r-\delta+\frac{\sigma^2}{2}\right)\cdot T\right]$$
$$= \frac{1}{1\cdot\sqrt{1}}\left[\ln\left(\frac{100}{100}\right) + \frac{1}{2}\right] = \frac{1}{2}$$

$$\Rightarrow \quad d_2(S(0),0) = \frac{1}{2} - 1 = -\frac{1}{2}$$

$$\Rightarrow \Delta_{GC}(S(0),0) = N\left(\tfrac{1}{2}\right) - 30\cdot\frac{1}{\sqrt{2\pi}}\cdot e^{-\frac{\left(-\frac{1}{2}\right)^2}{2}}\cdot\frac{1}{1\cdot\sqrt{1}\cdot 100}$$
$$= 0.6915 - 30\cdot\frac{1}{100\sqrt{2\pi}}\cdot e^{-\frac{1}{8}}$$
$$= 0.5858 \;\blacksquare$$

46. You are to price options on a futures contract. The movements of the futures price are modeled by a binomial tree. You are given:

(i) Each period is 6 months.

(ii) $u/d = 4/3$, where u is one plus the rate of gain on the futures price if it goes up, and d is one plus the rate of loss if it goes down.

(iii) The risk-neutral probability of an up move is 1/3.

(iv) The initial futures price is 80.

(v) The continuously compounded risk-free interest rate is 5%.

Let C_I be the price of a 1-year 85-strike European call option on the futures contract, and C_{II} be the price of an otherwise identical American call option.

Determine $C_{II} - C_I$.

(A) 0

(B) 0.022

(C) 0.044

(D) 0.066

(E) 0.088

47. Several months ago, an investor sold 100 units of a one-year European call option on a nondividend-paying stock. She immediately delta-hedged the commitment with shares of the stock, but has not ever re-balanced her portfolio. She now decides to close out all positions.

You are given the following information:

(i) The risk-free interest rate is constant.

(ii)

	Several months ago	Now
Stock price	\$40.00	\$50.00
Call option price	\$ 8.88	\$14.42
Put option price	\$ 1.63	\$ 0.25
Call option delta	0.794	

The put option in the table above is a European option on the same stock and with the same strike price and expiration date as the call option.

5.

Calculate her profit.

=

Payoff − FV(Initial cost)

(A)	\$11
(B)	\$24
(C)	\$126
(D)	\$217
(E)	\$240

0 — When the options were written

t — "several months" That's the time @ which we calculate the profit.

T=1 — Exercise date

48. DELETED

49. You use the usual method in McDonald and the following information to construct a one-period binomial tree for modeling the price movements of a nondividend-paying stock. (The tree is sometimes called a forward tree).

(i) The period is 3 months.

(ii) The initial stock price is \$100.

(iii) The stock's volatility is 30%.

(iv) The continuously compounded risk-free interest rate is 4%.

At the beginning of the period, an investor owns an American put option on the stock. The option expires at the end of the period.

Determine the smallest integer-valued strike price for which an investor will exercise the put option at the beginning of the period.

(A)	114
(B)	115
(C)	116
(D)	117
(E)	118

$\Rightarrow$ Profit (@ time t) = Wealth @ time t $- FV_{0,t}$(Initial Cost)

- <u>Initial Cost</u>: $-100 \cdot v_c(S(0),0) + \Delta_c(S(0),0) \cdot S(0) \cdot 100$

$= 100(-8.88 + 0.794 \cdot 40)$

$= 2{,}288.00$

- <u>Wealth @ time t</u>:

$-100 \cdot v_c(S(t),t) + 100 \cdot \Delta_c(S(0),0) \cdot S(t)$

$= 100(-14.42 + 0.794 \cdot 50)$

$= 2{,}528.00$

$$\Rightarrow \boxed{\text{Profit(@ time } t) = 2{,}528.00 - 2{,}288.00 \cdot e^{r \cdot t}}$$

- We will get e^{rt} from <u>put-call parity</u>:

At time 0: $v_c(S(0),0) - v_p(S(0),0) = F^P_{0,T}(S) - Ke^{-r \cdot T}$

$8.88 - 1.63 = 40 - Ke^{-r \cdot T}$ (I)

At time t: $v_c(S(t),t) - v_p(S(t),t) = F^P_{t,T}(S) - Ke^{-r(T-t)}$

$14.42 - 0.26 = 50 - Ke^{-r(T-t)}$ (II)

(I) $\Rightarrow$ $Ke^{-r \cdot T} = 40 - 7.25 = 32.75$

(II) $\Rightarrow$ $Ke^{-r(T-t)} = 50 - 14.16 = 35.84$

Dividing the two eq'ns:

$$\frac{\cancel{K}e^{-r(\cancel{T}-t)}}{\cancel{Ke^{-rT}}} = \frac{35.84}{32.75} \Rightarrow e^{r \cdot t} = 1.0944$$

$\Rightarrow$ Profit(@ time t) $= 2{,}528 - 2{,}288 \cdot 1.0944 \cong 24$ ■

Delta-hedger's profit over a "small" time period

SET UP:

1st An agent writes an option @ time 0.

2nd The agent Δ-hedges @ time 0 by trading in the shares of the underlying stock.

$\Rightarrow$ The wealth is going to be:

@ time 0 : $$w(S(0),0) = -\underbrace{v(S(0),0)}_{\substack{\text{value of}\\ \text{the option}\\ \text{they wrote}}} + \underbrace{\Delta(S(0),0)}_{\substack{\Delta \text{ of}\\ \text{the}\\ \text{option}}} \cdot S(0)$$

Look @ the value of the portfolio, i.e., the agent's wealth at time dt (before the agent has rebalanced the portfolio to re-establish the Δ-hedge):

$$w(S(dt),dt) = -\boxed{v(S(dt),dt)} + \Delta(S(0),0) \cdot S(dt)$$

For a small time interval dt, we can use the $\Delta \cdot \Gamma \cdot \Theta$ approximation for the option's price, i.e.,

$$\begin{aligned} v(S(dt),dt) \cong\ & v(S(0),0) \\ & + \Delta(S(0),0) \cdot dS \\ & + \frac{1}{2}\Gamma(S(0),0)(dS)^2 \\ & + \Theta(S(0),0)\, dt \end{aligned}$$

$\Rightarrow$ The approximate time-dt wealth:

$$\begin{aligned} w(S(dt),dt) = & -\big(v(S(0),0) + \cancel{\Delta(S(0),0)dS} + \frac{1}{2}\Gamma(S(0),0)(dS)^2 \\ & + \Theta(S(0),0)\,dt\big) + \Delta(S(0),0) \cdot \underbrace{S(dt)}_{S(0)+\cancel{dS}} \end{aligned}$$