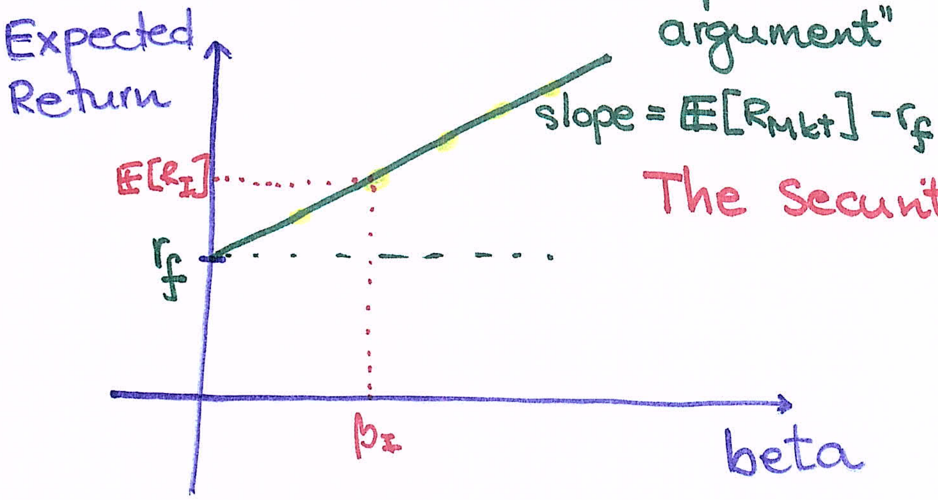
The Equity cost of Capital

W: April 29th, 2019.

Recall: In the CAPM: for any investment I

$$\mathbb{E}[R_I] = r_I = \underbrace{r_f}_{\text{the intercept}} + \beta_I \underbrace{\left(\overbrace{\mathbb{E}[R_{Mkt}] - r_f}^{\text{independent of the investment I}}\right)}_{\text{the slope}}$$

β_I: "independent argument"



The Security Market Line (SML)

Beta Estimation.

Linear Regression.

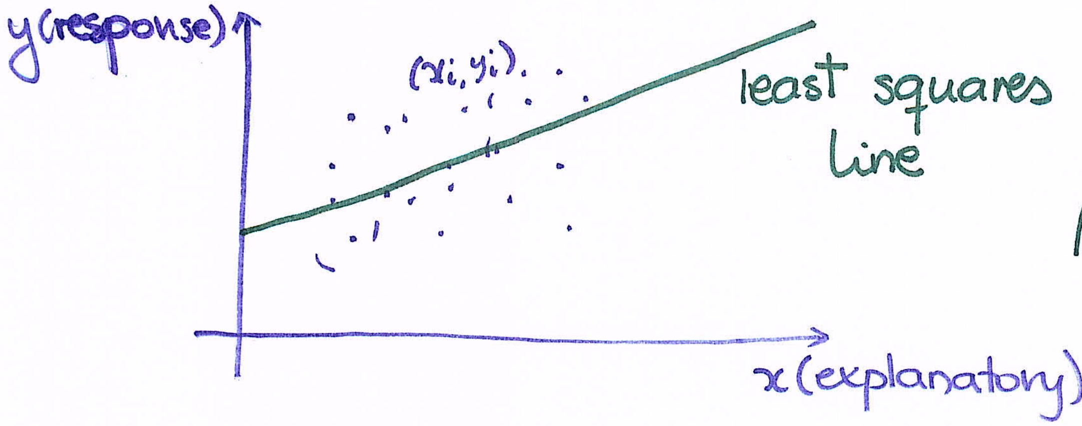
Explanatory random variable: X

Response random variable: Y

Model:
$$Y = \underset{\text{intercept}}{\alpha} + \underset{\text{slope}}{\beta} \cdot X + \varepsilon \sim N(0, \underbrace{\text{variance}}_{\text{assume to be the same for every value of } X}) \quad \text{(LR)}$$

Observed values:

(x_i, y_i), $i = 1..n$



$$\hat{\beta} = \frac{SD[Y]}{SD[X]} \cdot Corr[X,Y]$$

"Attacking" the LR w/ the expectation, we get

$$\mathbb{E}[Y] = \underbrace{\alpha + \beta}_{\text{estimated from the least squares!}} \cdot \mathbb{E}[X] + \cancel{0}$$

<u>In our applications</u>,

$$\underbrace{R_I - r_f}_{\text{the response variable}} = \alpha_I + \beta_I \underbrace{(R_{Mkt} - r_f)}_{\text{the explanatory variable}} + \varepsilon_I$$

the intercept of the linear regression

the slope of the linear regression

the error term

The expectation of the above:

$$\mathbb{E}[R_I] - r_f = \alpha_I + \beta_I(\mathbb{E}[R_{Mkt}] - r_f) + \overbrace{\mathbb{E}[\varepsilon_I]}^{=0}$$

$$\mathbb{E}[R_I] = \underbrace{r_f + \beta_I(\mathbb{E}[R_{Mkt}] - r_f)}_{\text{the security market line (SML)}} + \underbrace{\alpha_I}_{\text{the distance from the SML, i.e., the stock's alpha}}$$

13) The following table shows the beta and expected return for each of five stocks.

Stock (i)	β_i	$E(R_i)$
1	1.2	0.124
2	1.0	0.110
3	0.7	0.103
4	0.4	0.068
5	0.1	0.047

All of these stocks except one lie on the Security Market Line.

Calculate the alpha of the stock that does NOT lie on the Security Market Line.

A) –0.026

B) –0.014

C) 0.000

D) 0.014

E) 0.026

• There is no obvious efficient way to tackle this problem :(

⇒ Start w/ the first two stocks assuming they both lie on the SML:

$$\mathbb{E}[R_1] = r_f + \beta_1(\mathbb{E}[R_{Mkt}] - r_f)$$
$$\mathbb{E}[R_2] = r_f + \beta_2(\mathbb{E}[R_{Mkt}] - r_f)$$

⇒ From the table:

$$0.124 = r_f + 1.2(\mathbb{E}[R_{Mkt}] - r_f) \quad / -$$
$$-\;0.110 = r_f + 1\cdot(\mathbb{E}[R_{Mkt}] - r_f)$$

$$0.014 = 0.2(\mathbb{E}[R_{Mkt}] - r_f)$$

$$\Rightarrow \mathbb{E}[R_{Mkt}] - r_f = 5 \cdot 0.014 = 0.07$$

3.

$\Rightarrow \quad r_f = 0.11 - 0.07 = 0.04$

The above slope & intercept are my candidates for the parameters of the SML.
Now, we verify if Stocks #3, #4, #5 lie on it.

* Checking for Stock #3:

$$0.103 \stackrel{?}{=} 0.04 + 0.7 \cdot 0.07 = 0.04 + 0.049 = 0.089 \quad \times$$

* Checking for Stock #4:

$$0.068 \stackrel{?}{=} 0.04 + 0.4 \cdot 0.07 = 0.04 + 0.028 = 0.068 \quad \checkmark$$

$\Rightarrow$ Stock #4 is on the SML.

$\Rightarrow$ Stock #3 is not on the SML

$\Rightarrow \quad \alpha_3 = 0.103 - 0.089 = 0.014 \Rightarrow$ (D)