11.4. Risk vs. Return: Choosing an Efficient Portfolio

Efficient Portfolios contain <u>only</u> systematic risk.

↓

No other portfolio offers a higher expected return w/ lower or the same volatility.

<u>Inefficient portfolios</u>: It is possible to find another portfolio which is better in terms of expected returns and volatility.

04/30/2018.

6) You are given the following information about the four distinct portfolios:

Portfolio	Expected Return	Volatility
P	3%	10%
Q	5%	10%
R	5%	15%
S	7%	20%

Determine which two of the four given portfolios are NOT efficient.

(A) P and Q

(B) P and R

(C) P and S

(D) Q and R

(E) Q and S

. Note $P \prec Q$ (since $\sigma_P = \sigma_Q$ expected returns)

. —"— $R \prec Q$ (since $\sigma_R > \sigma_Q$ and the returns are the same!)

(B).

2.

The Effect of Correlation

- If $\rho = 1$, then we get a straight line between the two assets as the feasible set.
- If $\rho_{X,Y} = -1$, then ...

  $\boxed{\text{claim:}}$ There is a weight w such that the portfolio when w is the weight given to asset X has volatility zero.

$$Var[wR_X + (1-w)R_Y] = 0$$

$$\Leftrightarrow$$

$$w^2 \underbrace{Var[R_X]}_{\sigma_X^2} + (1-w)^2 \cdot \underbrace{Var[R_Y]}_{\sigma_Y^2} + 2w(1-w)\underbrace{Cov[R_X, R_Y]}_{\rho_{X,Y} \cdot \sigma_X \cdot \sigma_Y} = 0$$

$$\Leftrightarrow \quad w^2\sigma_X^2 - 2w(1-w)\sigma_X\sigma_Y + (1-w)^2 \cdot \sigma_Y^2 = 0$$

$$\Leftrightarrow \quad (w\sigma_X - (1-w)\sigma_Y)^2 = 0$$

$$\Leftrightarrow \quad w\sigma_X - (1-w)\sigma_Y = 0$$

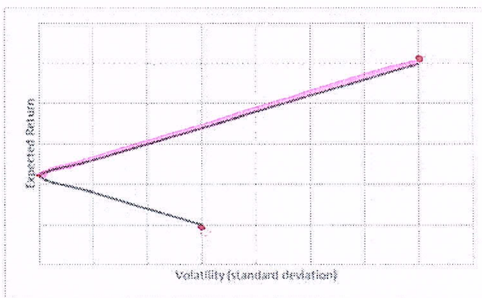
$$\Leftrightarrow \quad w(\sigma_X + \sigma_Y) = \sigma_Y \quad \Leftrightarrow \quad \boxed{w = \frac{\sigma_Y}{\sigma_X + \sigma_Y}}$$

- The higher the correlation, the smaller the curvature of the feasible set.

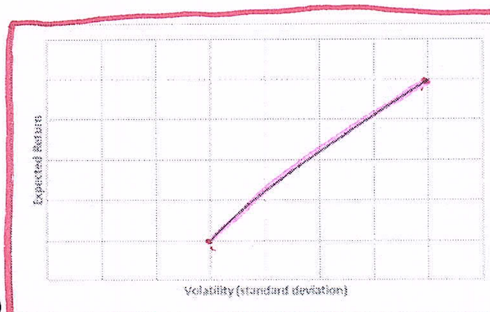
X and Y

5) You are given the following set of diagrams for a two-stock portfolio, with expected return on the vertical axis and volatility on the horizontal axis. These diagrams are meant to help investors identify the set of efficient portfolios.

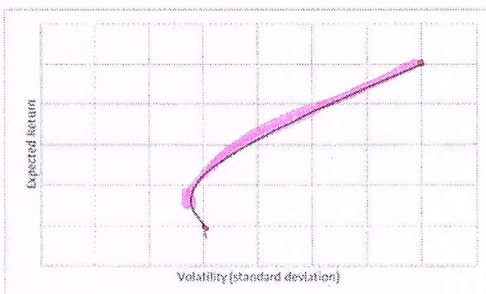
Identify the diagram demonstrating the highest correlation between the two stocks.



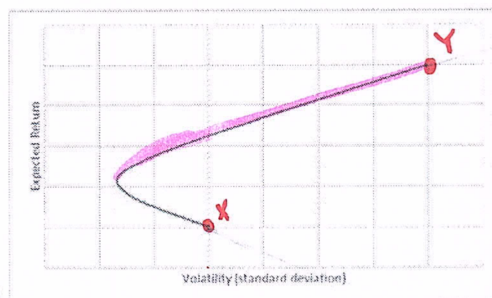
(A)



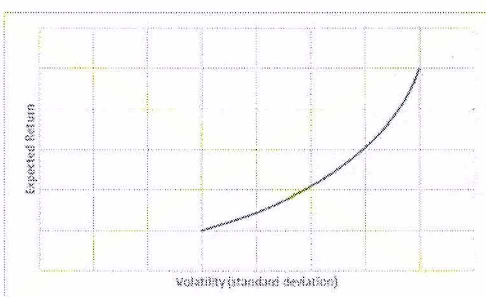
(B)



(C)



(D)

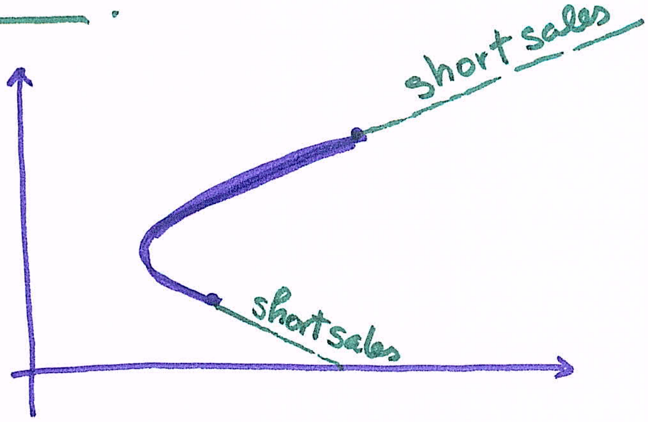


(E)

Not a feasible set!

4.

Short Sales



Efficient Portfolios w/ More stocks

Q: What is the shape of the feasible set?

Say, we have 3 assets X, Y, Z.

Parametrize the feasible set using pairs w_X, w_Y of weights given to X and Y. Note: the Z's weight is $1 - w_X - w_Y$

All of the feasible portfolios returns are of the form:

$$w_X \cdot R_X + w_Y \cdot R_Y + (1 - w_X - w_Y) \cdot R_Z$$

11.5. Risk-free saving & borrowing

So far: We looked @ portfolios of risky investments.
Say, we include all the available risky investments in constructing the efficient frontier, i.e., we have diversified as much as possible.
Another way to reduce risk is to invest @ the risk-free interest rate.

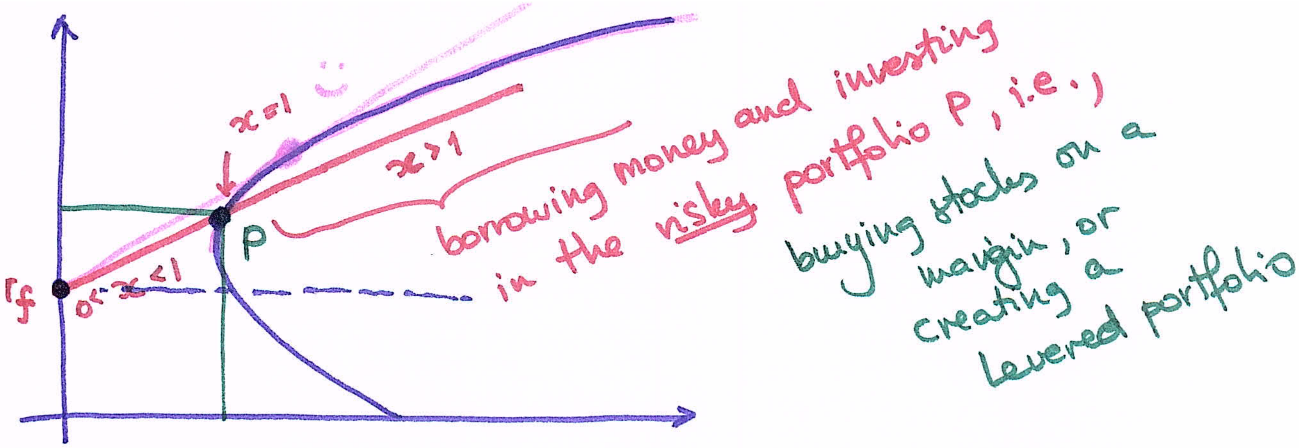
Say) You start w/ a ~~risk-free~~ risky portfolio P. Its return is denoted by R_P.
Let the risk-free interest rate be denoted by r_f.
Look @ a porfolio w/
weight x given to the risky P
and
weight $(1-x)$ given to the risk-free investment.
Let the return of the new portfolio be R_{xP}

$$\Rightarrow \mathbb{E}[R_{xP}] = x \cdot \mathbb{E}[R_P] + (1-x) \cdot r_f$$
$$= r_f + x(\mathbb{E}[R_P] - r_f)$$

$$\Rightarrow \mathbb{E}[R_{xP}] - r_f = x(\mathbb{E}[R_P] - r_f)$$

Also: $\mathrm{Var}[R_{xP}] = \mathrm{Var}[x \cdot R_P + (1-x) \cdot r_f]$
$$= \mathrm{Var}[x \cdot R_P] = x^2 \cdot \mathrm{Var}[R_P]$$

$$\Rightarrow \boxed{SD[R_{xP}] = x \cdot SD[R_P] = x \cdot \sigma_P}$$



What is the slope of the line through $(0, r_f)$ and $(\sigma_P, \mathbb{E}[R_P])$?

i.e., the line representing all the portfolios built out of P and the risk-free investment?

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\mathbb{E}[R_P] - r_f}{\sigma_P} =: \text{Sharpe Ratio}$$

$$\| $$

$$\frac{\text{Portfolio's excess Return}}{\text{Portfolio's volatility}} = \left(\frac{\text{Reward}}{\text{Risk}} \text{ ratio} \right)$$