The Capital Asset Pricing Model (CAPM).

w: April 26th, 2019.

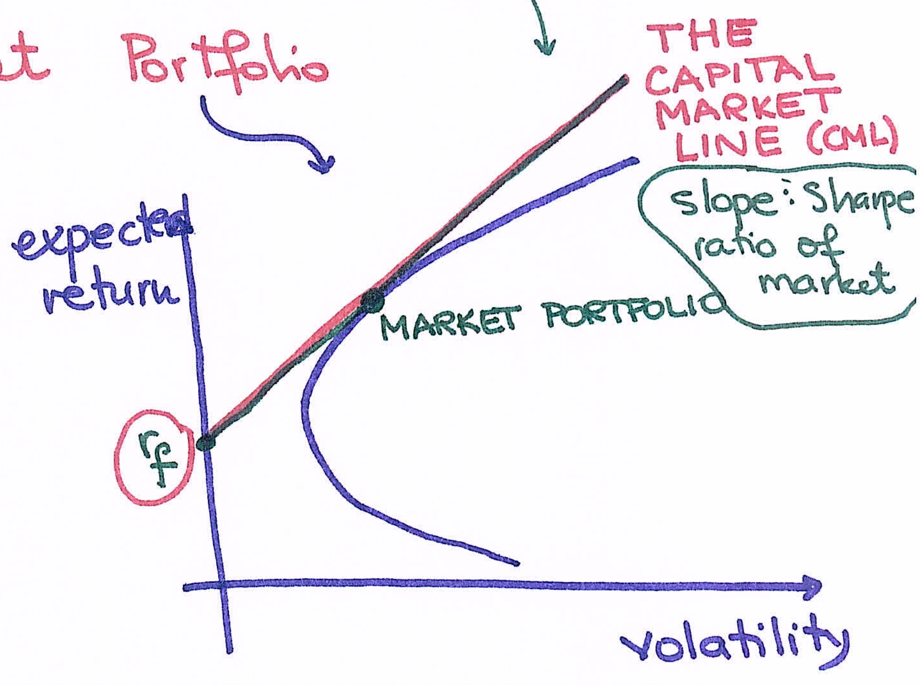
Assumptions.

1. No friction.
2. Rational behavior ⇒ Risk aversion.
   ⇒ Agents hold only efficient portfolios.
3. HOMOGENEOUS EXPECTATIONS.

⇓

Efficient Portfolio = Tangent Portfolio

= Market Portfolio



All assets i:

- # of shares of i
- market price per share

MV_i... market value of asset i:

MV_i = # shares × price

⇒ x_i... weight in the Market Portfolio

$$x_i = \frac{MV_i}{\sum MV_i}$$

Starting w/ a portfolio P and considering an investment I:

$\boxed{\text{required return}}$

$$\mathbb{E}[R_I] > r_I = r_f + \beta_I^P(\mathbb{E}[R_P] - r_f)$$

If P is **efficient**:

$$\boxed{\mathbb{E}[R_I] = r_I = r_f + \beta_I^{eff}(\mathbb{E}[R_{eff}] - r_f)}$$

$\Rightarrow$ In CAPM:

$$\mathbb{E}[R_I] = r_I = r_f + \underbrace{\beta_I}_{=:\beta_I^{Mkt}}(\mathbb{E}[R_{Mkt}] - r_f)$$

w/ $$\beta_I = \frac{SD[R_I]}{SD[Mkt]} \cdot corr[R_I, R_{Mkt}]$$

or

$$\beta_I = \frac{Cov[R_I, R_{Mkt}]}{Var[R_{Mkt}]}$$

16) You are given the following information about Stock X and the market:

(i) The annual effective risk-free rate is 5%. $r_f = 0.05$

(ii) The expected return and volatility for Stock X and the market are shown in the table below:

	Expected Return	Volatility
Stock X	5% $= \mathbb{E}[R_x]$	40% $= \sigma_x$
Market	8% $= \mathbb{E}[R_{Mkt}]$	25% $= \sigma_{Mkt}$

(iii) The correlation between the returns of stock X and the market is –0.25.

$$\text{corr}[R_x, R_{Mkt}] = -0.25$$

Assume the Capital Asset Pricing Model holds. Calculate the required return for Stock X and determine if the investor should invest in Stock X.

(A) The required return is 1.8%, and the investor should invest in Stock X.

(B) The required return is 3.8%, and the investor should NOT invest in stock X.

(C) The required return is 3.8%, and the investor should invest in stock X.

(D) The required return is 6.2%, and the investor should NOT invest in Stock X.

(E) The required return is 6.2%, and the investor should invest in stock X.

$$\rightarrow: r_x = r_f + \beta_x(\mathbb{E}[R_{Mkt}] - r_f)$$

$$\text{w/ } \beta_x = \frac{\sigma_x}{\sigma_{Mkt}} \cdot \text{corr}(R_x, R_{Mkt}) = \frac{0.4}{0.25} \cdot (-0.25) = -0.4$$

$\Rightarrow$ The required return for X is

$$r_x = 0.05 + (-0.4)(0.08 - 0.05) = 0.038 < \mathbb{E}[R_x]$$

3.

$\Downarrow$

Should invest in X!

14) You are given the following information about Stock X, Stock Y, and the market:

(i) The annual effective risk-free rate is 4%.

(ii) The expected return and volatility for Stock X, Stock Y, and the market are shown in the table below:

	Expected Return	Volatility
Stock X	5.5%	40%
Stock Y	4.5%	35%
Market	6.0%	25%

(iii) The correlation between the returns of stock X and the market is –0.25.

(iv) The correlation between the returns of stock Y and the market is 0.30.

Assume the Capital Asset Pricing Model holds. Calculate the required returns for Stock X and Stock Y, and determine which of the two stocks an investor should choose.

(A) The required return for Stock X is 3.20%, the required return for Stock Y is 4.84%, and the investor should choose Stock X.

(B) The required return for Stock X is 3.20%, the required return for Stock Y is 4.84%, and the investor should choose Stock Y.

(C) The required return for Stock X is 4.80%, the required return for Stock Y is 4.84%, and the investor should choose Stock X.

(D) The required return for Stock X is 6.40%, the required return for Stock Y is 3.16%, and the investor should choose Stock Y.

(E) The required return for Stock X is 3.50%, the required return for Stock Y is 3.16%, and the investor should choose both Stock X and Stock Y.

4.

Key: A

For Stock X, $\beta = -0.25*(40\%/25\%) = -0.40$.

The required return is $4\% + \beta*(6\% - 4\%) = 4\% + (-0.40)(2\%) = 3.20\%$.

For Stock Y, $\beta = 0.30*(35\%/25\%) = 0.42$.

The required return is $4\% + \beta*(6\% - 4\%) = 4\% + (0.42)(2\%) = 4.84\%$.

The expected return for Stock X exceeds the required return, but the expected return for Stock Y is lower than its required return, so the investor should invest in stock X only.

Reference: Berk/DeMarzo, Section 11.6

5.

15) You are given the following information about Stock X, Stock Y and the market

(i) The expected return and volatility for Stock X, Stock Y and the market are shown in the table below:

	Required Return	Volatility
Stock X	3.0%	50%
Stock Y	?	35%
Market	$\underbrace{6.0\%}_{=\mathbb{E}[R_{Mkt}]}$	25%

(ii) The correlation between the returns of stock X and the market is –0.25. $= \rho_{X,Mkt}$

(iii) The correlation between the returns of stock Y and the market is 0.30. $= \rho_{Y,Mkt}$

Calculate the required return for Stock Y.

(A) 1.48%

(B) 2.52%

(C) 3.16%

(D) 4.84%

(E) 6.52%

<u>By</u> previous work:

$$r_Y = \underset{?}{r_f} + \beta_Y\left(\mathbb{E}[R_{Mkt}] - r_f\right) \quad (r_Y)$$

From the required return for Stock X:

$$r_X = r_f + \beta_X\left(\mathbb{E}[R_{Mkt}] - r_f\right)$$

$$0.03 = r_f + (-0.5)(0.06 - r_f)$$

$$\beta_X = \frac{\sigma_X \cdot \rho_{X,Mkt}}{\sigma_{Mkt}} = \frac{0.5(-0.25)}{0.25} = -0.5$$

$$\Rightarrow 1.5 r_f = 0.06 \quad \Rightarrow \quad \boxed{r_f = 0.04}$$

$$\overset{(r_Y)}{\Rightarrow} \quad r_Y = 0.04 + (0.42)(0.06 - 0.04) = 0.04 + 0.0084 = 0.0484$$

$$\beta_Y = \frac{\sigma_Y (\rho_{Y,Mkt})}{\sigma_{Mkt}} = \frac{0.35(0.3)}{0.25} = 0.42$$

6.

7) Consider a portfolio of four stocks as displayed in the following table:

Stock	Weight	Beta
1	0.1	1.3
2	0.2	-0.6
3	0.3	*X*
4	0.4	1.1

Assume the expected return of the portfolio is 0.12, the risk free rate is 0.05, and the market risk premium is 0.08.

Also, assume that all four stocks are subject to the Capital Asset Pricing Model.

For every investment:

$$\mathbb{E}[R_I] = r_I = r_f + \beta_I(\mathbb{E}[R_{Mkt}] - r_f)$$

Calculate *X*.

A) 0.80

B) 1.06

C) 1.42

D) 1.83

E) 2.17

From Stock 1:

$$\cdot\ \mathbb{E}[R_1] = r_1 = r_f + \beta_1(\underbrace{\mathbb{E}[R_{Mkt}] - r_f}_{\text{The Market's Risk Premium}})$$

$$= 0.05 + 1.3(0.08) = 0.154$$

$$\cdot\ \mathbb{E}[R_2] = 0.05 + (-0.6)(0.08) = 0.002$$

$$\cdot\ \mathbb{E}[R_4] = 0.05 + 1.1(0.08) = 0.138$$

$$\underset{\substack{\uparrow \\ \text{given}}}{0.12} = \mathbb{E}[R_P] = \sum_{i=1}^{4} w_i \cdot \mathbb{E}[R_i] = 0.1 \cdot 0.154 + 0.2 \cdot 0.002 + 0.3 \cdot \mathbb{E}[R_3] + 0.4 \cdot 0.138$$

$$\Rightarrow \mathbb{E}[R_3] = 0.1633 \underset{\substack{\uparrow \\ \text{CAPM}}}{=} r_f + \beta_3 \cdot (\mathbb{E}[R_{Mkt}] - r_f)$$

7.

$$\beta_3 = \frac{0.1633 - 0.05}{0.08} = 1.416 \Rightarrow (C)$$

Beta of a Portfolio

Start w/ some portfolio P.

Let $R_P = x_1 \cdot R_1 + \cdots + x_n \cdot R_n$

$$\begin{array}{ccc} \uparrow & & \uparrow \\ \beta_1 & \cdots & \beta_n \end{array}$$

$$\beta_P = \frac{\mathrm{Cov}[R_P, R_{Mkt}]}{\mathrm{Var}[R_{Mkt}]} = \frac{\mathrm{Cov}[x_1 \cdot R_1 + \cdots + x_n \cdot R_n, R_{Mkt}]}{\mathrm{Var}[R_{Mkt}]}$$

$$= \frac{\sum_{i=1}^{n} x_i \, \mathrm{Cov}[R_i, R_{Mkt}]}{\mathrm{Var}[R_{Mkt}]} = \sum_{i=1}^{n} x_i \beta_i$$