Problem 4.2. Assume the Black-Scholes framework. You are given the following information for a stock that pays dividends continuously at a rate proportional to its price:

(i) The current stock price is \$250. $= S(0)$

(ii) The stock's volatility is 0.3. $= \sigma$

(iii) The continuously compounded expected rate of stock-price appreciation is 15%.

$\downarrow$

$\alpha - \delta = 0.15$

Find the value s^* such that

$$\mathbb{P}[S(4) > s^*] = 0.05.$$

(a) \$861.65

(b) \$874.18

(c) \$889.94

(d) \$905.48

(e) None of the above.

$$S(T) = S(0)e^{(\alpha - \delta - \frac{\sigma^2}{2})\cdot T + \sigma\sqrt{T}\cdot Z}$$

w/ $Z \sim N(0,1)$

We know that the percentiles of the std normal Z correspond to the percentiles of the stock price $S(T)$.

So, we find the critical value z^* s.t.

$$\mathbb{P}[Z > z^*] = 0.05$$

Then,

$s^* = S(0)e^{(\alpha - \delta - \frac{\sigma^2}{2})\cdot T + \sigma\sqrt{T}\cdot z^*}$ w/ be the constant we seek

Our $z^* = 1.645$

$$\Rightarrow \quad s^* = 250e^{(0.15 - \frac{0.09}{2})\cdot 4 + 0.3\sqrt{4}\cdot(1.645)}$$

$$\Rightarrow \quad \boxed{s^* = 1020.92}$$

1.

Review:

$$\mathbb{P}[S(T) > K] = N(\hat{d}_2)$$

w/ $\hat{d}_2 = \frac{1}{\sigma\sqrt{T}}\left[\ln\left(\frac{S(0)}{K}\right) + (\alpha - \delta - \frac{\sigma^2}{2})\cdot T\right]$

Note: Under the risk-neutral measure $\mathbb{P}^*$,

$$\mathbb{P}^*[S(T) > K] = N(d_2)$$

w/ $d_2 = \frac{1}{\sigma\sqrt{T}}\left[\ln\left(\frac{S(0)}{K}\right) + (r - \delta - \frac{\sigma^2}{2})\cdot T\right]$

Q: What is the probab. that a European K-strike put is in the money on its exercise date T?

→: $\mathbb{P}[S(T) < K] = 1 - \mathbb{P}[S(T) \geq K]$

S(T) is continuous

$= 1 - N(\hat{d}_2)$

$= N(-\hat{d}_2)$

$$\boxed{\mathbb{P}[S(T) < K] = N(-\hat{d}_2)}$$

Problem. Let the current stock price be \$100.

The stock price @ any later date is modeled as lognormal.

According to your model:

$$\begin{cases} \cdot\ \mathbb{P}[S(1/4) < 95] = 0.2358 \\ \cdot\ \mathbb{P}[S(1/2) < 110] = 0.6026 \end{cases}$$

What is the expected value of the time·1 stock price?

→: For any T, we have

$$S(T) = S(0)\, e^{\underbrace{(\alpha - \delta - \frac{\sigma^2}{2})}_{=:\mu}\cdot T + \sigma\sqrt{T}\cdot Z}$$

w/ $Z \sim N(0,1)$

rate of appreciation

Recall: $\mathbb{E}[S(T)] = S(0)\, e^{(\alpha-\delta)\cdot T}$

$$\Rightarrow \mathbb{E}[S(1)] = S(0)\, e^{\mu + \frac{\sigma^2}{2}}$$

· Focus on $\mathbb{P}[S(1/4) < 95] = 0.2358$

The complementary probab: $1 - 0.2358 = 0.7642$

$$\Rightarrow z^*_{0.2358} = -0.72$$

$$\Rightarrow 95 = 100\, e^{\mu(1/4) + \sigma\sqrt{1/4}\cdot(-0.72)}$$

$$\Rightarrow \frac{1}{4}\mu + \frac{1}{2}\sigma(-0.72) = \ln(0.95) \quad (\mathrm{I})$$

· Focus on $\mathbb{P}[S(1/2) < 110] = 0.6026$

$$z^*_{0.6026} = 0.26$$

$$\Rightarrow \quad 110 = 100 \cdot e^{\mu(1/2) + \sigma\sqrt{1/2}\,(0.26)}$$

$$\Rightarrow \quad \frac{1}{2}\mu + \sigma\sqrt{1/2}\,(0.26) = \ln(1.1) \qquad (\mathrm{II})$$

Combine (I) & (II).

Get :

$$\sigma = 0.2189 \quad ; \quad \mu = 0.1101$$

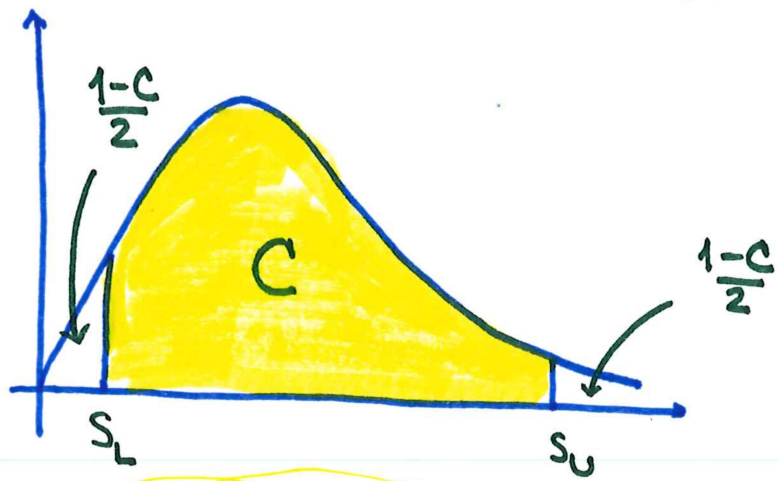
Finally, $\mathbb{E}[S(1)] = 100\, e^{0.1101 + \frac{(0.2189)^2}{2}}$

$$= 114.35 \quad \blacksquare$$

Log Normal "confidence" intervals.

By design: · two-sided
and
· symmetric

Given a probability, i.e., a "confidence" level $C \in (0,1)$



Let $z^* = N^{-1}\left(\frac{1+C}{2}\right)$.

Then,

$$S_U = S(0)\, e^{(\alpha - \delta - \frac{\sigma^2}{2})\cdot T + \sigma\sqrt{T}\cdot z^*}$$

$$S_L = S(0)\, e^{(\alpha - \delta - \frac{\sigma^2}{2})\cdot T + \sigma\sqrt{T}\,(-z^*)}$$

* **50.** Assume the Black-Scholes framework.

You are given the following information for a stock that pays dividends continuously at a rate proportional to its price.

(i) The current stock price is 0.25. $= S(0)$

(ii) The stock's volatility is 0.35. $= \sigma$

(iii) The continuously compounded expected rate of stock-price appreciation is 15%.
$\alpha - \delta$

Calculate the upper limit of the 90% lognormal confidence interval for the price of the stock in 6 months.

$T = 1/2$ $\qquad z^* = N^{-1}(0.95) = 1.645$

(A) 0.393

(B) 0.425

(C) 0.451

(D) 0.486

(E) 0.529

$\Rightarrow S_U = 0.25e^{\left(0.15 - \frac{(0.35)^2}{2}\right)\cdot\frac{1}{2} + 0.35\sqrt{\frac{1}{2}}\cdot(1.645)}$

$\Rightarrow S_U = 0.393 \Rightarrow$ (A) ■

51-53. DELETED

54. Assume the Black-Scholes framework. Consider two nondividend-paying stocks whose time-t prices are denoted by $S_1(t)$ and $S_2(t)$, respectively.

You are given:

(i) $S_1(0) = 10$ and $S_2(0) = 20$.

(ii) Stock 1's volatility is 0.18.

(iii) Stock 2's volatility is 0.25.

(iv) The correlation between the continuously compounded returns of the two stocks is –0.40.

(v) The continuously compounded risk-free interest rate is 5%.

(vi) A one-year European option with payoff $\max\{\min[2S_1(1), S_2(1)] - 17, 0\}$ has a current (time-0) price of 1.632.

Consider a European option that gives its holder the right to sell either two shares of Stock 1 or one share of Stock 2 at a price of 17 one year from now.

Calculate the current (time-0) price of this option.

6.